Multi-Object Auctions with Package Bidding: An Experimental Comparison of *i*BEA and Vickrey*

Yan Chen Kan Takeuchi

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Abstract

We study two package auction mechanisms in the laboratory, a sealed bid Vickrey auction and an ascending version of Vickrey called *i*BEA. Unlike the single-unit Vickrey where bidders tend to overbid in the laboratory, most of our bidders either underbid or bid their true values. At the aggregate level, Vickrey generates significantly higher revenue and efficiency than iBEA. We also find that human bidders learn from their robot opponents when the robot strategies are (myopic) best responses.

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Chen: School of Information, University of Michigan, 1075 Beal Avenue, Ann Arbor, MI 48109-2112. Email: yanchen@umich.edu. Takeuchi: Department of Economics, University of Michigan, 611 Tapan Street, Ann Arbor, MI 48109-1220; Email: ktakeuch@umich.edu.

1 Introduction

In recent years, the use of various multi-object auction mechanisms for resource allocation has been rapidly increasing. The Federal Communications Commission (FCC) spectrum auctions, characterized by synergies across licenses, stimulated tremendous research interests in complex auction design for multiple objects and with synergies. Beginning on July 25, 1994, the FCC has used the Simultaneous Multiple Round (SMR) auction to allocate spectrum and raised over $41 billion in revenue (Milgrom 2004). However, this auction format does not allow package bidding. Ledyard, Porter and Rangel (1997) demonstrate that the performance of the FCC design degrades in the presence of complementarities. More generally, when bidder valuations for multiple objects are super-additive, package bidding are necessary to increase efficiency, seller revenue and bidder willingness to participate (Bykowsky, Cull and Ledyard 2000). These studies help to spur interests in package auction designs at the FCC, and in complex resource allocation in general. For example, FCC auction No. 31 for selling spectrum licenses in the 700 MHz band, was designed to permit bids for any of the 4095 possible packages of the twelve licences on offer.[1]

The earliest package auction design is the proposal for a sealed bid combinatorial auction sale of paired airport takeoff and landing slots proposed by Rassenti, Smith and Bulfin (1982). Banks, Ledyard and Porter (1989) present two kinds of iterative package auctions, AUSM (Adaptive User Selection Mechanism) and the iterative Vickrey-Clarke-Groves mechanism (Vickrey (1961), Clarke (1971), Groves (1973)), for allocating uncertain and unresponsive resources. In laboratory experiments, these package auctions significantly outperforms markets and administrative procedures. More recently, Kwasnica, Ledyard, Porter and DeMartini (forthcoming) create and test a new design for multi-object iterative auctions by merging the better features of AUSM and the FCC SMR design. The resulting new Resource Allocation Design (RAD) is shown to perform better than both its parents.

Another important application of package auction design is Business-to-Business (B2B) auctions, which are predominantly multi-object and often involve synergies. Package auctions have the potential to provide value to both buyers and sellers of goods and services. The descending Dutch auction used to sell flowers in Aalsmeer, Holland, is a simple version of a package auction for homogeneous goods (Katok and Roth 2004). More recently, package auctions have been successfully applied to transportation procurement. For example, Sears Logistics Services is the first procurer of trucking services to use a package auction to reduce its costs. In 1993 and in subsequent auctions, it consistently saved 13 percent over past procurement practices, mostly bilateral negotiations (Ledyard, Olson, Porter, Swanson and Torma 2002). The London bus routes market provides an example of the use of package auction format in public procurement. In this instance, the local transportation authority has adopted a form of package auction because of expected economic synergies among routes located in the same area of London. The London bus routes auction has lead to increased quality of service and lower costs, and thus is considered a success (Cantillon and Pesendorfer 2006).

Despite the successful applications of various forms of package auctions, in most cases, the theoretical properties of these auctions are not completely understood. Therefore, it is unclear whether the actual choice of design options are the most appropriate ones.

[1] See http://wireless.fcc.gov/auctions/31/ for more information.

An important standard for nearly all mechanism design work, and for auctions in particular, is the Vickrey-Clarke-Groves (VCG) mechanisms. The VCG mechanism is dominant strategy incentive compatible, i.e., bidding one's true valuation is always optimal regardless of others' strategies. Furthermore, it implements the efficient outcome. For the single object case, VCG mechanism becomes the familiar second-price auction. In the auction context, we follow the convention and call the VCG mechanism Vickrey auctions. Despite its attractive theoretical properties, the Vickrey auction has some disadvantages. In the package auction context, there are three main concerns.[2] First, it might be vulnerable to collusion. For example, bidders can use shill bidders to increase competition in order to generate lower prices. Second, it might suffer from the monotonicity problem,[3] i.e., adding bidders might reduce equilibrium revenues. Third, previous laboratory experiments show that, in the single object case, the dominant strategy in second-price auctions is not transparent. Many experimental subjects consistently overbid in second-price auctions and they do not seem to learn (see, e.g., Kagel (1995)). Contrary to economists' belief that Vickrey auctions are rarely used in practice, Lucking-Reiley (2000b) presents evidence that Vickrey auctions have long been the predominant auction format for mail sales of collectible postage stamps, at least 65 years earlier than the publication of Vickrey's seminal paper. Vickrey-like auctions have also been appearing in auctions on the Internet, sometimes with an additional feature of "proxy bidding." In these auctions, a bidder tells his proxy his maximum willingness to pay. The proxy keeps this information secret and bid on the bidder's behalf in an ascending auction with pre-announced increment. If every bidder uses a proxy, then the bidder with the highest maximum price wins and pays (approximately) the second highest price. The most prominent examples include Amazon and eBay.[4]

Because of the above-mentioned disadvantages of the sealed-bid Vickrey auctions, there have been considerable efforts to search for an ascending bid package auction with comparable theoretical properties. The Ausubel-Milgrom Ascending Proxy auctions (Ausubel and Milgrom 2002) replicates the performance of the Vickrey auction when all goods are substitutes, and has full information equilibrium outcomes that are bidder-optimal points in the core. Parkes and Ungar (2002) present the iBundle Extend & Adjust (*i*BEA) auction, an ascending bid auction with package bidding which implements efficient allocation and Vickrey payments in *ex post* Nash equilibrium, with only a free-disposal requirement on agent preferences. Thus, theoretically, *i*BEA represents a major advance in modelling package auctions.

In this paper, we investigate two package auctions in the laboratory, *i*BEA and Vickrey, to evaluate their performance among boundedly rational individuals. Compared to previous experimental studies of package auctions, the theoretical properties for these two auctions are well understood. In particular, this is the first experimental study of *i*BEA.

We study these two mechanisms in a simple environment where three bidders compete for four items (and thus 15 packages), with synergies across subsets of the items. Each human bidder competes against two automated bidders. In half of the treatments, the

[2] Note the first two problems do not appear when all goods are substitutes for all bidders.

[3] See Milgrom (2004) Chapter 8 for examples.

[4] Ockenfels and Roth (2002) analyze the closing rules of eBay and Amazon. They show that eBay auctions with a hard closing rule, give bidders incentives for sniping, while Amazon auctions, with a soft closing rule, does not give such incentives and hence is a more faithful dynamic version of Vickrey.

automated bidders are programmed to follow the dominant strategy in Vickrey, and Myopic Best Response in *i*BEA, while in the other half of the treatments, automated bidders are programmed to follow the random, or zero-intelligence strategies. The use of automated agents serve two purposes. First, it allows the experimenter to compare the performance of the two mechanisms in an environment free from the strategic uncertainties inherent in interactions between human bidders.[5] Second, the use of automated agents is becoming increasingly widespread in Internet auctions, which allows for conveniently asynchronous bidding (Lucking-Reiley 2000a). It is therefore, important to study how humans react when they bid against automated agents.

The main results are surprising. Contrary to previous experimental studies of sealed bid vs. ascending bid auctions, where ascending auctions tend to outperform their sealed bid counterpart, in our study, the sealed bid Vickrey auctions generate significantly higher efficiency and revenue than *i*BEA. Unlike in second-price auctions where most participants overbid, most participant in Vickrey package auctions either underbid or bid their true values. We also find that human bidders learn from automated agents with "intelligent" strategies.

Section 2 introduces the auction mechanisms. Section 3 presents the experimental design. Section 4 presents the hypotheses. Section 5 presents the analysis and main results. Section 6 concludes.

2 The Auctions

We introduce the two auction mechanisms in this section. We first set up a simple framework that allows us to explain the auctions in a precise and fairly easy manner.

Let $N = \{1, \ldots, n\}$ be a finite set of bidders. Let i denote an agent, where $i = 0$ is an *auctioneer*, and $i > 0$ is a bidder. $N_0 = N \cup \{0\}$ is the set of all bidders and the auctioneer. Let $K = \{1, \ldots, k\}$ represent the set of objects to be sold, and $X = \{0, 1\}^k$ represent the set of combinations of objects. Let $\mathcal{B}_i$ be a set of *bids* of bidder i. Each bid is a pair (x, p), where $x \in X$ corresponds to the packages desired and $p \in \mathbb{R}$ is the bid price. Let $v_i : X \rightarrow \mathbb{R}_+$ be bidder i's *valuation function* that assigns a value to a package. Finally, $\delta = (\delta_1, \ldots, \delta_b)$ is an indicator vector whose elements are 1 or 0, where δ_j indicates whether bid (x_j, p_j) is *winning* or *losing*, and where b is the total number of bids.

As the winner determination problem is part of every package auction design, we formally define it.

Definition (Package auction winner determination problem). The package auction winner determination problem is to maximize the sum of bids, indicating each bid as *winning* or *losing*, under the constraint that each item can be sold to at most one bidder;

$$\max_{\delta} \sum_{j=1}^{b} \delta_j p_j \quad \text{subject to} \quad \sum_{j \in \{j : \delta_j = 1\}} x_j \leq (1, 1, \ldots, 1). \tag{1}$$

[5] See, e.g., Kagel and Levin (2001) for an experiment with human bidders against automated agents in multi-unit auctions of homogeneous goods.

To achieve one of the most important goals in multi-object auctions, we define an important concept as follows: a bid $\langle \mathbf{x}, p \rangle \in \mathcal{B}_i$ is a *truthful bid*, if $v_i(\mathbf{x}) = p$. Then, the fundamental theorem of welfare implies that if every bidder bids for all packages and if all of the bids are truthful, then the allocation associated with the solution to the winner determination problem is Pareto efficient.

From the view point of the auctioneer and/or mechanism designer, therefore, it is desirable that all bidders truthfully bid for all packages. However, in theory, it is not always optimal for a bidder to bid her true value, and in practice, it is not easy for bidders to calculate all values for all possible packages. Thus, to design an auction mechanism properly, we would like to have the most efficient allocation among feasible outcomes. There are three major problems to overcome when designing a multi-object auction mechanism.

1. The exposure problem: When items are not substitutes and bidders cannot bid on packages, bidders are usually exposed to the risk that they might end up with overpaying. For example, suppose that there are two items and that bidder i has strong complementarity such that $v_i((1,0)) = v_i((0,1)) = 1 < 3 = v_i((1,1))$. He may bid on each item at prices more than 1, expecting that he gets both of the items. However, it is possible that he could get only one item paying more than 1. Auctions with package bidding, such as the two auctions studied in this paper, should overcome this problem.
2. The threshold problem. Suppose that there are 4 bidders and 3 items to trade and that $v_1((1,0,0)) = v_2((0,1,0)) = v_3((0,0,1)) = 1.5$ and $v_4((1,1,1)) = 4$. It is efficient to allocate the items to Bidders 1, 2 and 3. However, suppose that Bidders 1, 2 and 3 bid on their desirable item at price 1, respectively and that Bidder 4 bids on the package of all items at price 3.6. In this case, the winning bid is Bidder 4's bid, but none of Bidder 1-3 can overbid on Bidder 4's bid. The ascending bid auction, *i*BEA, overcomes the threshold problem, which we will explain in Section 2.
3. Computational problems. There are two computational problems, one of which is the auctioneer's problem and the other lies on the bidders' side. For the auctioneer, the problem is to solve Eq. (1), which is NP-complete.[6] For the bidders, they are required to evaluate all possible combinations, which can be cognitively difficult. Evaluation of the latter is where experimental research can be especially valuable.

In what follows, we introduce the Vickrey and *i*BEA auctions, and discuss their theoretical properties.

2.1 Vickrey Auction with Package Bidding

Vickrey auction with package bidding is an extension of the more familiar second-price auction. At the beginning of each auction, each bidder selects which packages he would

[6]In computational complexity theory, the class P consists of all those decision problems that can be solved on a deterministic sequential machine in an amount of time that is polynomial in the size of the input; the class NP consists of all those decision problems whose positive solutions can be verified in polynomial time given the right information, or equivalently, whose solution can be found in polynomial time on a non-deterministic machine. Informally, the NP-complete problems are the "toughest" problems in NP in the sense that they are the ones most likely not to be in P. (http://en.wikipedia.org/wiki/) However, relying on some existing computer softwares, practically it is not always unsolvable.

like to bid on, and how much he would like to bid for each of those packages. Each bidder can choose to bid on as many packages as he wants, and he can bid on a single object multiple times, by bidding on several packages that contain that item. However, no matter how many packages a bidder bids on, he will never win more than one package. This type of bidding is called the exclusive-or (XOR) bids. Note that XOR bids are necessary for truth-telling to be a dominant strategy in Vickrey.

Once all bidders have submitted their bids, the auctioneer will choose the combination of submitted bids that yields the highest sum of bids. The set of bidders winning a package are the winning bidders.

The auctioneer will then, one at a time, choose each of the winning bidders as a **pivotal bidder**. The auctioneer examines the bids again, but ignores the bids of the pivotal bidder. The auctioneer determines the allocation of goods that maximizes the sum of bids, using the same rules as before, but not considering any bids placed by the pivotal bidder. Once this new allocation has been determined, the auctioneer compares the sum of bids generated by this allocation with those generated when no bids were excluded.

The amount that the winning bidders are required to pay at the end of the auction depends on the additional revenue that each bidder generated, which is calculated by comparing the original revenue obtained by the auctioneer, versus the revenue obtained by the auctioneer when the given bidder was pivotal. The following example from the experimental instruction illustrates how it works.

Example: Suppose that there are three bidders and four objects to allocate, and the following bids are submitted:

	Package	Price	status
Bidder 1	**AB**	£50	winning
Bidder 2	**CD**	£40	winning
Bidder 3	**ABCD**	£60	
Bidder 3	**AB**	£30	

As shown in the table, the bids from Bidder 1 and 2 are the winning bids, because they generate the highest revenue £50 + £40 = £90.

However, the auctioneer does not ask Bidder 1 to pay £50. Suppose that Bidder 1 were chosen as a pivotal bidder, and so his bids are ignored. The winning bids then become Bidder 2's bid on **CD** and Bidder 3's bid on **AB**.

	Package	Price	status
~~Bidder 1~~	~~**AB**~~	~~£50~~	~~winning~~
Bidder 2*	**CD**	£40	winning
Bidder 3	**ABCD**	£60	
Bidder 3*	**AB**	£30	winning

Then the auctioneer calculates the revenue that those winning bids would generate, which is £40 + £30 = £70. Thus, the additional revenue that Bidder 1 generates is £20, since £90 – £70 = £20. This £20 is the price adjustment for Bidder 1. Therefore, Bidder 1 pays £50 and receives £20 back. His final price is £30.

It is well known that **Vickrey auction** is dominant strategy incentive compatible and implements an efficient outcome, where each bidder i ends up with Vickrey payoff in equilibrium. However, it has some shortcomings, as we discussed in the Introduction. The first disadvantage is its vulnerability to collusion. It is an empirical question whether bidders can successfully collude. Second, the revenue under Vickrey can be very low. The third weakness is that the dominant strategy in Vickrey might not be transparent when it is implemented among boundedly rational people. Based on many previous experimental studies on single unit auctions (see Kagel (1995) for a survey) where bidders systematically overbid in single-unit Vickrey auctions, we expect the ascending bid auctions to achieve higher efficiency than the Vickrey auction in the multi-object setting with package bidding.

2.2 *i*Bundle Extend & Adjust (*i*BEA)

The *i*Bundle Extend & Adjust (*i*BEA) auction is proposed by Parkes and Ungar (2002). It is an ascending-price generalized Vickrey auction. It maintains non-linear and non-anonymous prices on packages, and terminates with approximately efficient allocation and the Vickrey payments. To achieve these properties, the mechanism requires myopic best response, which is an *ex post* Nash equilibrium. Each auction takes place in several rounds. Let ε be the price increment. The choice of ε involves a tradeoff between the speed of the auction and the closeness to efficiency of the final outcome. That is, a smaller ε can achieve closer to efficient outcomes at the cost of a longer auction.

1. Initialize prices for all packages.
2. At the beginning of each round, for each bidder, the auctioneer announces "ask prices" $\mathbf{p}_i(t)$ for all packages to bidder i.
3. Given the prices, each bidder can bid on as many packages as she wants, and she can bid on a single item multiple times, by bidding on several packages that contains that item. Each bidder's submitted bids, $\mathcal{B}_i(t)$, must satisfy the following rules:
   (a) Winning bid resubmission rule: $\exists(x, p) \in \mathcal{B}_i(t)$ such that (x, p) is a winning bid at the previous round. That is, if a bidder has made a winning bid in a previous round, she is obligated to bid on that package, at the same price, in the next round. Once a bid is losing, a bidder has no further commitment to bid on that package, unless he chooses to.
   (b) Last-and-final bid: The last-and-final option allow a bidder to continue to bid for a package while the bid price is narrowly above its value. Once a bidder chooses the last-and-final option on a package, he gets a small discount, ε, and the last-and-final bid will be automatically resubmitted at the same price in every round until the auction terminates. Therefore, even if the price for that package increases, a bidder cannot increase his bid on that package.

       The last-and-final option facilitates the computation of Vickrey prices. It also reveals to the auctioneer a bidder's approximate true value for a package.
4. If there are no new bids, then auction terminates. Otherwise, given $\{\mathcal{B}_i(t)\}_{i\in N}$, the auctioneer solves Eq. (1) and revises ask prices to each bidder in the following manner:

(a) The ask prices for a bidder with any winning bid(s) remain the same as the previous round.

(b) The prices for packages with last-and-final bids remain the same as the previous round.

(c) The price for each package of a losing bid increases by ε

(d) Adjust all ask prices so that they are self-consistent, i.e., the price for a package should not be less than the price for any of its subset.

Going to the next round $t + 1$, then go back to step 2.

There are two phases to determine an auction's outcome. The auctioneer uses Phase I to determine the final allocation and Phase II to compute the final prices and Vickrey discounts. Phase I terminates when all agents who submit bids are assigned a package, and the allocation will be the final allocation. The auction proceeds to Phase II. In each round of Phase II, the auctioneer selects a pivotal bidder and ignores his bids to compute his *externality*. When she computes all externalities, the prices is determined, from which she computes Vickrey payoffs, in the same manner as in the sealed bid Vickrey.

Just as Ausubel and Milgrom (2002) rely on the assumption of straightforward bidding strategy, in *i*BEA it is assumed that all bidders take a *Myopic Best-Response* (MBR). We say bidder i takes myopic best-response to ask prices $\mathbf{p}_i(t)$ announced by the auctioneer, if

$$\mathcal{B}_i(t) = \left\{ (x, p) \middle| v_i(x, p) - p \geq \max \left\{ \max_{x \in X} \{v_i(x, p(x)) - p(x)\}, 0 \right\} - \varepsilon. \right\} \tag{2}$$

where $p(x)$ is the ask price of package x for bidder i. MBR chooses ε-maximized packages, i.e., packages arbitrarily close to the best package.

We now explain MBR in more detail. We define a bidder's *temporary profit* as $v_i(x, p(x)) - p(x)$. Therefore, when a bidder examines his menu, he first looks for any packages for which he has a negative temporary profit. For those packages that have a negative profit, he adds ε to his temporary profit, because he knows that he can buy the package with a ε discount using the last-and-final option. He does not change his temporary profits for packages for which he currently has a positive profit. He then examines the revised temporary profits for all packages, and finds the package that gives him the greatest revised temporary profit. There are two possibilities:

1) If his greatest possible profit is greater than or equal to ε, he will bid on all packages that give him a revised temporary profit of within ε of the maximum temporary profit. For example, if one package gives him a temporary profit of £17, and no package gives him a profit of more than £17, and $\varepsilon = \pounds 5$ then he will bid on all packages that give him a temporary profit of at least £17 – £5, or £12.
2) If his greatest possible profit is less than ε, he will make bids only on those packages that have a revised temporary profit of greater than or equal to 0.

Under the assumption of MBR, we can achieve competitive equilibrium prices and efficiency. More precisely, if bidders follow MBR, then the vector of ask prices $\{\mathbf{p}_i(t)\}_{i \in N}$ is a competitive equilibrium price vector after the end of Phase I. As $\varepsilon \to 0$, and the final allocation is arbitrarily close to the efficient allocation. Furthermore, Phase II allows the auctioneer to compute Vickrey payoffs. Parkes and Ungar (2002) also prove that MBR is incentive compatible: MBR is an ex-post Nash equilibrium of *i*BEA, as $\varepsilon \to 0$.

To illustrate how *i*BEA works, we use a simple 3-bidder, 2-item example. In this example, we assume that all bidders follow MBR strategy. Furthermore, we set $\epsilon = 5$.

[Table 1 about here.]

Table 1 illustrates how *i*BEA works. The top panel presents phases I and II round-by-round and the bottom panel compares the result with the Vickrey auction.

As shown in the table, for Bidder 1 the value of **A**, **B** and **AB** are (10, 0, 10). For Bidder 2 and 3, they are (0, 30, 30) and (4, 14, 22), respectively.

At the first round, the offered prices are all same, the auctioneer breaks the tie by randomly choosing as many winning bidders as possible, and Bidder 1 and 2 are selected. Since Bidder 3 does not win any package, the auctioneer raises the offered prices for Bidder 3. Specifically the price for the losing bid on **AB** is raised by the increment of 5. In Round 2, the highest temporary profit of Bidder 3 is $17 = 22 - 5$ and that Bidder 3, following MBR, is willing to bid on any package whose temporary profit is above $12 = 17 - 5$. Bidder 3 makes bids on not only (**AB**, 5) but also (**B**,0), since the temporary profits of those packages are 17 and 14. The auctioneer chooses (**AB**,5) from Bidder 3 as the winning bid for Round 2. The process continues. Notice, however, that Bidder 3 makes Last-and-Final bids on (**A**, 5) and (**B**,15) in Round 8, because the offered prices are above the value of those packages. Then, these bids are submitted with the discount of 5 and become ***(A, 0)*** and ***(B,10)***.

At the end of Round 9 in which there is no new bid, the allocation of items is finalized: Bidder 1 receives **A** and Bidder 2 receives **B**. The auction proceeds to Phase II. Before going to the next round, there should be one or more round(s) which is not observable to the bidders. The auctioneer randomly chooses one of the winners as a pivotal bidder, which is Bidder 2 in this example. Then, the auctioneer excludes Bidder 2's bids from the bids submitted in Round 9 and selects the new set of wining bids ignoring Bidder 2's bids. The new winning bid is ***(AB, 20)*** from Bidder 3. So, the auctioneer raises the offered prices for Bidder 1 and proceeds to Round 10. In Round 10, (**A**,10) from Bidder 1 and (**B**, 10) from Bidder 3 are selected as the winning bids, and there is no new bid again. Choosing Bidder 1 as the next pivotal bidder and the auctioneer goes through the similar process above. At the end of Round 11, since there is no new bid and no winner for the next pivotal bidder, the auction terminates.

After that, the auctioneer determines the price adjustment (rebate) for Bidder 1 and 2. Note that the revenue is 30 before any price adjustment. Based on the offered prices at the end of the auction, the auctioneer calculates the additional revenue that each of the winners generates. When Bidder 1 is pivotal, Bidder 2 receives **B** and Bidder 3 receives **A** and the revenue would be 20. Thus, the additional revenue from Bidder 1 is $10 = 30 - 20$, and this 10 is the price adjustment for Bidder 1. Similarly, the price adjustment for Bidder 2 is 10. All payoff information is summarized in the bottom panel.

To our best knowledge, *i*BEA has more desirable properties than any other ascending package auctions proposed so far, but it has not been tested in the laboratory. Past experiments on single-unit auctions show that the ascending bid auction, e.g, English clock auction, achieves higher efficiency than the sealed bid Vickrey auction, even though the solution concept is weaker (Kagel 1995). The strength of the ascending auction lies in its feedbacks, which make the optimal strategy more transparent than Vickrey. Therefore, it is interesting to see whether this superior performance of the ascending bid auction carries over to the multi-object package bidding context.

3 Experimental Design

Our experimental design reflects both theoretical and technical considerations. Specifically, we are interested in three important questions. First, the comparative performance of two potentially useful mechanisms, *i*BEA and Vickrey. Second, human subjects' response to the degree of rationality in the environment. Third, whether human subjects imitate various bidding strategies of their robot opponents. We describe the environment and the experimental procedures below.

3.1 The Economic Environment

In each auction, three bidders compete for 4 items (**A**, **B**, **C** and **D**), and thus 15 packages.[7] Each subject competes against two automated bidders, i.e., computer programs (hereafter, robots). In all sessions, subjects were informed that they interacts with robots. We choose to use robots rather than other human bidders as using robots gives us more control over each human subject's environment. For the human participants, this environment reduces the strategic uncertainties inherent in interactions between human bidders. Furthermore, it allows us to observe human subject reactions when they interact with robots. The latter in itself, has important implications for e-commerce (Eisenberg n.d.).

We implemented a $2 \times 3 \times 2$ design. In the first dimension, we compare 2 mechanisms, i.e., *i*BEA and Vickrey. In the second dimension, we implemented 3 different combinations of strategies for the robots. As Sincere bidding (S) leads to efficiency allocation in both mechanisms, while Random bidding (R) represents zero-intelligence, we set up 3 different different combinations of these strategies, i.e., SS, SR and RR. Namely, both of the robots follow Sincere bidding in SS, one follows Sincere and the other follows Random in SR, and both follow Random in RR. As we are interested in whether a subject would imitate a certain strategy when they are told that one of the robots follow such a strategy, we designed a third dimension with two information conditions. In the low information treatments, subjects are told that they are competing against robots, however, robots strategies are not explained to the subjects. In the high information condition, we explain the robot strategies in the instructions.[8]

We now describe bidder preferences. Let V_i be the value of package i. The value for each item is drawn independently from a uniform distribution on $\{0, 1, 2, \cdots, 10\}$. The value of a package is the sum of the values of the items in the package, plus the bonus value for certain combinations of items due to synergy. A human bidder and the first robot derive synergy from **A** and **B**. In the experiment, we choose a CES function to represent this

[7] These packages are **A**, **B**, **C**, **D**, **AB**, **AC**, **AD**, **BC**, **BD**, **CD**, **ABC**, **ABD**, **ACD**, **BCD**, and **ABCD**.

[8] Instructions are lengthy, therefore, rather than putting them in the appendix, we post them on: http://www.si.umich.edu/~yanchen/

type of preference. Thus, the value of A and B together equals $V_{AB} = (V_A^\rho + V_B^\rho)^{1/\rho}$. The CES function allows us to control for the degree of complementarity and substitutability. When $\rho > 1$, $V_{AB} < V_A + V_B$. When $\rho = 1$, $V_{AB} = V_A + V_B$. When $0 < \rho < 1$, $V_{AB} > V_A + V_B$, i.e., **A** and **B** have synergy. In the experiment, we choose $\rho = 0.9$, as we are interested in the case when synergy is present. Similarly, the second robot derives synergy from **C** and **D**. For example, if a human bidder or robot 1 has package **ABCD**, her value equals $V_{ABCD} = V_{AB} + V_C + V_D$. If robot 2 has the same package, its value equals $V_{ABCD} = V_A + V_B + V_{CD}$. To summarize, the human bidder and robot 1 has the same preferences, and thus the environment is more competitive for them than for robot 2.

For *i*BEA, we set $\varepsilon = \pounds 5$, as the smallest grid size is £1, and $\varepsilon = \pounds 5$ generates a reasonable speed of convergence in the lab. When we analyze the experimental results in Section 5, we will compare the performance of the two mechanisms using the actual grid size of $\varepsilon = \pounds 5$ for *i*BEA and $\varepsilon = \pounds 1$ for Vickrey, as well as the same grid size of $\varepsilon = \pounds 5$ for both auctions by using simulations.

3.2 Experimental Procedures

Our experiment involves 8 to 10 players per session. At the beginning of each session, each subject is given printed instructions. After the instructions are read aloud, subjects are encouraged to ask questions. The instruction period take on average 21 minutes for *i*BEA and 14 minutes for Vickrey. Then subjects take a quiz designed to test their understanding of the mechanisms. At the end of the quiz, the experimenters go through the answers in public. The quiz takes an average of 17 minutes for *i*BEA and 11 minutes for Vickrey. At the end of the quiz, subjects randomly draw a PC terminal number. Each then sits in front of the corresponding terminal and starts the experiment.

In each session, each subject participates in 10 auctions. As we are interested in learning, there are no practice auctions. At the beginning of each auction, the value for each item is randomly drawn from a uniform distribution on $\{0, 1, 2, \cdots, 10\}$ for each bidder.

In the Vickrey treatments, each subject is informed of the value of the 15 packages on his screen. He can enter an integer bid for any of the 15 packages.[9] Note that a zero bid on a package is treated differently from no bid. A subject can be allocated a package which he bids zero, but can never be allocated a package that he does not bid on. Meanwhile, each robot submits a bid on each package, following the pre-assigned strategy. A robot following the Sincere strategy bids its true value for each package; while a robot following the Random strategy randomly chooses a number between 0 and 120% of its value for each package. The upper bound is chosen based on previous experimental evidence on single-unit Vickrey auctions (Kagel 1995). The server collects all bids from each group, compute the final allocation and payoffs for each bidder and send this information back to his screen. Each human bidder gets the following information at the end of each auction: his allocation, his price and price adjustments, his value for the allocated package, his profit, his cumulative profit.

In the *i*BEA treatments, each subject is given a menu, which contains the following columns: package, his value for each package, price, temporary profit, and two check boxes

[9]The zTree program put a lower bound of zero, and an upper bound of 1000 for the bids.

(whether he wants to bid on a package, and whether he wants his bid to be the last-and-final bid), the status of his previous bid (winning, losing, last-and-final and winning, and last-and-final and losing). He checks either, both or neither of the checkboxes. Recall that for winning bids and last-and-final bids, the checkboxes are automatically checked.

Meanwhile, a robot following the Sincere strategy adopts MBR. A robot following the Random strategy examines his menu and looks for packages for which he has an original temporary profit of greater than −£5. For those packages that do, he flips a coin to decide whether or not to place a bid on that package.

Both types of robots will choose the Last-and-Final option for any packages they place bids on that have an original temporary profit which is negative.

Then the auction proceeds as described in Section 2.

[Table 2 about here.]

Table 2 presents features of the experimental sessions, including mechanisms, Robot strategy profiles, information conditions, the shorthand notation for each treatment, the number of subjects for each treatment, as well as the exchange rates.[10] Overall, 12 independent computerized sessions were conducted in the RCGD lab at the University of Michigan from July to November 2003. We used zTree (Fischbacher 1999) to program our experiments. Our subjects were students from the University of Michigan.[11] No subject was used in more than one session, yielding 115 subjects. Each *i*BEA session lasted approximately two hours, while each Vickrey session lasted approximately one hour. We provide incentives for the quiz questions. A subject with fully correct answer gets $5 in addition to their auction earnings. Each mistake in the quiz costs 50 cents. The average earning (including quiz award) was $26.6 for Vickrey and $47.13 for *i*BEA. Data are available from the authors upon request.

4 Hypotheses

Based on the theoretical predictions and our experimental design, we identify the following hypotheses.

Hypothesis 1. *In Vickrey auctions, bidders will bid on all packages.*

Hypothesis 2. *In Vickrey auctions, bidders will bid truthfully.*

Hypotheses 1 and 2 are based on the dominant strategy of the Vickrey auction. The next two hypotheses are based on the theoretical predictions of *i*BEA.

Hypothesis 3. *In iBEA, bidders will bid on packages within £5 of the maximum temporary profit.*

[10] At the end of iSS_ℓ, iSS_h and iSR_ℓ, the actual earnings of subjects were low, therefore, we adjust the exchange rate to $1 equal £1 for iSS_ℓ and iSS_h, and $1 equal £1.2 for iSR_ℓ. Since these adjustments happened at the end of the experiment, and no subject was used for more than one session, we do not believe that the adjustments affected the results.

[11] Doctoral students in Economics are excluded from participation.

Hypothesis 4. *In iBEA, bidders will choose the last-and-final option for all packages with negative temporary profits.*

We now formulate hypotheses which compare the performance of the two mechanisms. As *i*BEA is an ascending version of Vickrey, we expect, *ex ante*, that the two mechanisms will generate the same bidder profit, auctioneer revenue and efficiency.

Hypothesis 5. *Vickrey and iBEA will generate the same amount of bidder profit.*

Hypothesis 6. *Vickrey and iBEA will generate the same amount of auctioneer revenue.*

Hypothesis 7. *Vickrey and iBEA will generate the same efficiency.*

As the environment is increasingly competitive with an increase in the number of Sincere robots, we expect that the human bidder will have the highest profit when competing against two Random Robots, and will have the lowest profit when competing against two Sincere Robots. We also expect the human bidder to learn from robot strategies in the high information treatments.

5 Results

In this section, we will first examine individual behavior under Vickrey and *i*BEA. We then compare the aggregate performance of the two auctions.

5.1 Individual Behavior in Vickrey

For the Vickrey mechanism, we have 60 subjects, each of whom independently plays 10 auctions with two robots in one of the six environments. In each auction, a subject can bid on any of the 15 packages at any price between £0 and £1000.

Unlike the single item case, the strategy in a multi-item Vickrey auction with package bidding has two dimensions. The first is whether to bid on a package. The second is how much to bid on a package if one decides to bid on it.

[Figure 1 about here.]

Figure 1 presents simulated profits for the human bidder under the three different environments in Vickrey. The top panel (a) is the environment with two sincere robots. The middle panel (b) is the environment with one sincere and one random robot. The bottom panel (c) is the environment with two random robots. The horizontal plane consists of two dimensions of strategies, the probability of bidding on a package, and the bid/value ratio for a package. We generate 10,000 hypothetical auctions, each of which consists of independent draws of the values for items A, B, C and D from the uniform distribution on $\{0, 1, 2, \cdots, 10\}$. The preferences for the human and robot bidders, and the rules of the auction are identical to the experimental environment. For each combination of the Probability of Bidding (drawn from the interval $[0, 1]$ with a grid size of 0.02) and Bid/Value ratio (drawn from the interval $[0, 2]$ with a grid size of 0.04), we compute the average profit for the human bidder across these 10,000 auctions, and report it on the vertical axis. Comparing the profit from each strategy combination across environments, as expected, Sincere-Sincere is the most competitive environment, with the lowest profit level for the human bidder at any given combination of the probability of bidding and the bid/value ratio, followed by Sincere-Random, which in turn is followed by Random-Random. To analyze the tradeoffs within an environment, we use the contour set of these figures.

[Figure 2 about here.]

Figure 2 presents the contour sets of the three environments of Figure 1. The horizontal axis is the Bid/Value ratio, while the vertical axis is the Probability of Bidding. Each curve represents combinations of the bid/value ratio and the probability of bidding which yields the same profit. The highest profit is achieved in each case when the probability of bidding is 1, and the bid/value ratio is also 1, i.e., bidding on every package and bidding one's true value for each package, which is the dominant strategy. Each curve is similar to an indifference curve. The "inner" (or "upper") curves represents high profit levels.

[Table 3 about here.]

Table 3 presents human bidders' bidding decisions averaged across all treatments. The Active Bids column presents the ratio of positive and zero bids over 600, which is the total number of bids if every bidder bids on every package. Note a zero bid is an active bid, while no bid is not active. The Bid/Value ratio is the mean ratio of the bid to the value of a package. The next three columns present the proportion of Truthful Bidding, Overbidding and Underbidding among active bids.

Even though bidding on every package is a weakly dominant strategy, the second column of Table 3 shows that the proportion of bids never reaches 100% on any package. Furthermore, participants bid on packages containing items AB more frequently. We now investigate which factors induce higher proportion of active bids. We use a probit model with robust clustering at the individual level. The dependent variable is Active Bids, a dummy variable, which equals 1 if a bidder places an active bid on a package and zero otherwise. The independent variables include Value of a package, a dummy variable D_{AB}, which equals one if a package contains both items A and B, and zero otherwise, and a dummy variable D_{HS}, which equals one if the information condition is High and there is at least one sincere robot bidder, and zero otherwise. The reason for including the latter is that participants might learn from the sincere robot strategy. Even though we are explicit in the instructions for all High Information treatments that robots bid on all packages, participants might ignore both parts of the random robot strategy, as the second part is obviously not optimal. Indeed, replacing the last dummy with a dummy D_H, which equals one with High Information and zero otherwise, results in insignificant coefficient.

Result 1 (Whether to Bid on a Package). Bidders are significantly more likely to bid on packages with higher values. In addition, they are more likely to bid on packages with synergetic items. The proportion of active bids increases significantly in treatments with at least one sincere robots under high information.

[Table 4 about here.]

Support. Table 4 presents results from the two probit specifications described above. Coefficients are probability derivatives. The Value of a package increases the likelihood of bidding on this package by 0.8%. If a package contains the synergistic bundle AB, the likelihood that a subject bids on this package is increased by 8.5%. Compared to other information conditions, subjects increase the likelihood of bidding on packages by 16.4%. All coefficients are significant at the one- or five-percent level. ∎

Result 1 indicates that bidders are significantly more likely to bid on high value packages and those with synergistic bundles. Furthermore, bidders imitate the Sincere Robot, but not the Random Robot.

To investigate how much participants bid in Vickrey, we use a structural approach based on Hypothesis 2, which is based on the fact that bidding one's true valuation is a weakly dominant strategy. To test this hypothesis, we use an OLS regression with clustering at the individual level. In the first specification, we use Bid as the dependent variable, and Value as the only independent variable. We do not include a constant because of the theoretical prediction. In the second specification, we add Auction, Cumulative Profit, D_{AB} and D_{HS} as independent variables. The variable Auction denotes the number of auctions in a session, thus it captures the learning effect, if any. For each specification, we run two-sided Wald tests of the null hypothesis of bids being equal to values against the alternative hypothesis of bids not equal to values. Results are presented in Table 5.

[Table 5 about here.]

Table 5 indicates that, on average, bidders underbid, rather than overbid. In specification (1), the coefficient for Value is 0.962, which is rather close to truthful preference revelation. To classify bidders, we repeat the first specification in Table 5 for each bidder. We then perform the Wald test for the null hypothesis that the coefficient on `Value` is 1 and subsequently classify bidders into the following groups.

1. Under-Bidder: If we can reject the hypothesis of truthful bidding at the 5% level and the coefficient is below 1.

   Underbidding: 69.83% overall; 1st auction: 73.33%; last auction: 78.33%.

2. Truthful Bidder: If we cannot reject the hypothesis at the 5% level.

   Truthful Bids: 7.83% overall, 1st round: 11.67%; last round: 3.33%.

3. Over-Bidder: If we can reject the hypothesis at the 5% level and the coefficient is above 1.

   Over-bids: 22% overall; 1st auction: 15%; last auction: 16.67%.

We now summarize the analysis of bidding behavior in Vickrey in the following result.

Result 2 (Bid Price in Vickrey). Bidders in Vickrey on average bid 96.2% of their true value. Bidders can be classified into three categories: Under-(57%), Truthful (32%) and Over-bidders (12%).

Support. Table 5 presents the OLS regression results for Vickrey auctions. The coefficient estimates show how much subjects bid compared to their valuations. Robust standard errors in parentheses are clustered at the individual level. Two-sided Wald test of the null hypothesis of bids being equal to values is 0.193. The classification of bidders comes from regressions at the individual level. The average R^2 of individual regressions is 0.934, with a standard deviation of 0.131. ∎

Most previous laboratory studies of single-unit Vickrey auctions consistently find that bidders tend to overbid (Kagel 1995). In multi-unit uniform price auctions, bidders tend to overbid on the first unit and underbid on the second unit, which is consistent with the theoretical prediction of demand reduction (Kagel and Levin 2001). Our finding that most bidders either underbid or bid their true value in Vickrey auctions is in stark contrast with previous experimental results. This is the first empirical evidence that the "robust" finding of overbidding in single-unit Vickrey does not carry over to the package Vickrey auction.

To investigate whether bidders learn from prior experience and how that experience affects bidder behavior, we do the following analysis. We first define *bidding pattern* as a combination of two different measures, the Number of Active Bids[12] in an auction and the Bid/Value Ratio. To identify the effects of prior experience while minimizing individual-specific characteristics on bidding behavior, we take a difference-in-difference approach. First, for each auction, we take the difference of the Number of Active Bids between the current and the previous auctions. We then classify all observations into two groups, a winner group where subject(s) won a package in the previous auction, and a loser group. Finally, we compare the difference between the two groups. We can analyze the Bid/Value Ratio in a similar way.

Result 3 (Effect of Prior Experience on Bidding). Losers in a previous auction are significantly more likely to change the number of active bids, compared to winners. Losers increase their bid/value ratio, while winners decrease their bid/value ratio. The difference is significant.

Support. A loser in a previous auction changes the number of active bids by 2.12 on average (with a standard error of 0.221), while a winner in the previous auction changes his number of active bids by only 1.22 (with a standard error of .116). The difference is statistically significant at the level of 1%. A two-sample t-test with equal variances yields a p-value of 0.0001.
A loser in a previous auction increases the bid/value ratio by 0.102 on average (with a standard error of 0.031), while winner in the previous auction decreases his bid/value ratio by 0.047 (with a standard error of .039). The difference is statistically significant at the level of 1%. A two-sample t-test with equal variances yields a p-value of 0.0084. ∎

Result 3 indicates that participants learn from prior experience. The directions of change for winners and losers are intuitive, and yet at the same time, indicating that the dominant strategy is not transparent to a substantial fraction of the participants.

To understand the individual learning dynamics in Vickrey, we also investigate individual learning by using two simple learning models, the reinforcement learning model (Erev and

[12] This is equivalent to the likelihood of bidding when the total number of packages are fixed.

Roth 1998) and the payoff assessment learning model (Sarin and Vahid 1999). While both models have been shown to tract human learning behavior fairly well both in relatively simple games (Erev and Roth 1998), games with complete information (Chen and Gazzale 2004) and of limited information (Chen and Khoroshilov (2003), Sarin and Vahid (2004)), neither tract the dynamics well in the more complex Vickrey auction. Results are available from the authors upon request.

5.2 Individual Behavior in *i*BEA

At any round of an *i*BEA auction, each participant strategy has two components: which package(s) to bid on, and when to check the Last-and-Final option.

We define *adjusted temporary profit* as the temporary profit of a package, after a subject consider whether to check its last-and-final option. For packages with the last-and-final option checked, the temporary profit is increased by £5. The myopic best response (MBR) strategy in *i*BEA states that bidders should bid on packages within ε of the maximum adjusted temporary profit. Among packages a participant can actively bid on,[13] we group them by their adjusted temporary profits in each round of the auction. Those within £5 of the maximum adjusted temporary profit are called the *MBR packages.* Sincere Robots only bid on MBR packages. The rest of the packages are called *non-MBR packages.*

[Figure 3 about here.]

Figure 3 presents the likelihood of human bidders to bid on MBR and non-MBR packages among all packages a participant can actively bid on, in each of the six different treatments. Comparing the low and high information treatments within SS, SR and RR, the proportion of bids (MBR and non-MBR) increase. However, the increase seems to be mostly on MBR packages in SS and SR, while in RR, the proportion of bids on both MBR and non-MBR packages have increased. This is consistent with the hypothesis that humans learn from robot strategies. We now use probit models to formally check our impression from Figure 3.

[Table 6 about here.]

Table 6 presents six probit specifications which examines factors affecting the likelihood of MBR bids. In all specifications, the dependent variable is PlaceBid, a dummy variable which equals one if a bid is placed on a package and zero otherwise. The independent variables are Temporary Profit, D_{ab}, D_{info} in specifications (1), (3) and (5). In specifications (2), (4) and (6), we add an additional independent variable, D_{info}*MBR, which equals 1 if the information condition is high and the package belongs to the set of MBR packages. This variable controls for the information condition on MBR package bidding. Note that D_{ab} and D_{info}*MBR are positively correlated. We summarize the results below.

[13] Recall that last-and-final packages and winning packages from previous rounds are automatically checked in the current round, therefore, a participant cannot do anything active about them. We therefore, do not consider them as part of the choice set for the subject.

Result 4 (Bidding Decision in *i*BEA). In *i*BEA, bidders are significantly more likely to bid on packages with higher temporary profit. Additional information on robot strategies
(1) induces significantly more bids on both MBR and non-MBR packages under RR, but not under SS or SR;
(2) induces significantly more bids on MBR packages under SS and SR, but not RR.

Support. In Table 6, coefficients on Temporary Profit is positive and significant in all six specifications. Coefficient on D_{info} is positive and significant in specification (5), but not in (1) - (4) and (6). The coefficient of D_{info}*MBR is positive and significant in (2) and (4), but not in (6). ∎

Result 4 confirms our intuition from Figure 3 that the knowledge of robot strategies significantly impact human bidding behavior. While Sincere Robot induces more MBR bids, Random Robot tends to increase bids on both types of packages. This suggests that in this complex auction, participants might not be able to figure out the optimal bidding strategy, and thus is susceptible to teaching. Therefore, in the actual implementation of *i*BEA, it might be a good idea to teach the participants MBR bidding.

In all treatments, all robots follow the equilibrium strategy in checking the Last-and-Final option among the set of packages they bid on, i.e., they check this option for a package whose temporary profit is negative. In the high information treatments, subjects are explicitly told when robots check the Last-and-Final option. We are interested in two questions. First, do subjects learn to use this option? Second, when they use this option, do they use it at the right time?

[Table 7 about here.]

Table 7 presents statistics on the mean temporary profit of all last-and-final bids in a treatment, the number of such bids where the temporary profits are negative, non-negative, the total number, as well as the proportion of negative last-and-final bids. The average temporary profit is £1.96 for all treatments, £0.77 for high information treatments, and £4.40 for the low information treatments. The difference between the high and low information treatments is highly significant over all, as well as within each environment (p-value < 0.01 for two sample t-tests with equal variance in all four cases). This indicates that subjects tend to check this option earlier than optimal, however, they learn from the robot strategies in the high information treatments. Another interesting result is that among the three high information treatments, the temporary profit of the last-and-final bids in the RR environment is significantly higher than that in the SS and SR treatments (p-value < 0.01 for two sample t-tests with equal variance). This could be due to two reasons. First, because RR is less competitive, the auction lasts fewer rounds than SR and SS. Therefore, SR and SS give bidders more time to learn. Second, the first dimension of a random robot's strategy (which packages to bid on) is obviously not optimal, that might have spillover effects on the second dimension, i.e., when to check the last-and-final option. To evaluate these causes, we use the following specifications.

[Table 8 about here.]

Table 8 reports the results of two OLS specifications, both of which controls for clustering at the individual level. In both specifications, the dependent variable is the Temporary Profit of the last-and-final bids. In specification (1), the independent variables are Round (the round when the last-and-final option is checked), D_{info} (a dummy variable which equals one under high information treatments and zero otherwise), and a constant. As we expect the temporary profit to be lower the longer an auction lasts, we use Round to control this effect, while we study the effects of information conditions. The coefficients of both Rounds and D_{info} are negative and significant. In specification (2), we add another independent variable, D_{HS}, a dummy which equals one if the information condition is high and there is at least one Sincere robot. When this variable is added, the coefficient for D_{info} is no longer significant at the 5% level. This indicates that the description of sincere robot strategies, rather than random robot strategies, is the driving force for adoption of strategies closer to the optimal.

5.3 Aggregate Performance of the Two Mechanisms

We examine the aggregate performance of the two mechanisms in three aspects: bidder profit, auctioneer revenue, and efficiency.

In examining bidder profit, we look at the human bidder profit as well as the aggregate profit of the human and robot bidders in each treatment. Auctioneer revenue follows the standard definition.

In single-unit auctions, efficiency is often measured by the ratio of the number of auctions where the object goes to the bidder with the highest valuation and the total number of auctions. In the multi-unit context, this definition is not applicable. We, instead, use the definition of efficiency developed by Kagel and Levin (2001). **Efficiency** of an auction is the ratio of the total surplus of the allocation and the highest possible surplus among all possible allocations, where total surplus is the sum of bidder profit and auctioneer revenue. In our environment, we have 256 possible allocations in total, as there are 4 items and 3 bidders ($256 = 4^4$). Note that it is not sufficient to consider $3^4 = 81$ allocations, since some allocations which leave item(s) unsold might yield the highest revenue to the auctioneer. For example, suppose that Bidder 1 bids only on A at \$1 and Bidder 2 bids only on B at \$1, and that Bidder 3 makes bids for C at \$2 and CD at \$1. In this case, the auctioneer does not sell D to anyone. Therefore, the problem is equivalent to allocating 4 items among 4 agents (3 bidders and an auctioneer), yielding 256 possible allocations.

[Figure 4 about here]

Figure 4 presents the distribution of actual observed efficiency in all auctions, pooling all treatments. The left panel is the efficiency distribution under *i*BEA, and the right panel is the efficiency distribution under Vickrey. It is clear that Vickrey on average generates higher efficiency.

[Table 9 about here.]

Table 9 presents the average human profit, total bidder profit, auctioneer revenue and efficiency under each auction mechanism. Recall that in the experiment, price increment under iBEA is £5 due to the length of the auction, while under Vickrey we use a grid size of £1. We take seriously the possibility that the performance of iBEA might be disadvantaged due to this difference in grid size. Therefore, under each category, we present two columns under Vickrey, one observed and the other hypothetical. Observed efficiency (or bidder profit or revenue) under Vickrey is the actual experimental observation using a grid size of £1, while the hypothetical measure uses a grid size of £5. In computing the latter, we round the actual bids up or down to the closest integer on a grid of size 5. In doing this, iBEA and Vickrey are more comparable in these measures. We of course do not rule out the possibility that participant behavior might in some way be psychologically affected by the different grid size, which is not captured by our hypothetical coarsification of the grid size in Vickrey. Comparing iBEA and the hypothetical Vickrey (as well as the actual Vickrey), we see that iBEA performs better in human profit as well as total profit, while Vickrey achieves higher Revenue and Efficiency. Since session average does not control for a number of exogenous factors, we use the following specifications to model factors affecting the performance of each mechanism.

[Table 10 about here.]

Table 10 presents four OLS specifications. The dependent variable in each specification is (1) Human Profit, (2) Total Profit, (3) Revenue, and (4) Efficiency. The independent variables are Mechanism, which equals 1 for iBEA and 2 for Vickrey, D_{info}, the number of random robots, quiz score and a constant. Note that we use a hypothetical grid of size 5 in computing each of these measures under Vickrey, so that it is comparable to iBEA. All results hold if we use the actual grid size of 1 under Vickrey. We summarize our findings below.

Result 5 (Bidder Profit). Human bidder profit as well as total profit are significantly higher under iBEA than under Vickrey. Furthermore, the number of random robots significantly increases human profit, and weakly increases total profit. A higher quiz score significantly increases human profit.

Support. In Table 10, for specification (1), the coefficient for Mechanism is negative and significant. The coefficients for the number of random robots and quiz score are both positive and significant. In specification (2), the coefficient for Mechanism is negative and significant. The coefficient for the number of random robots is weakly significant. ∎

By Result 5, we reject Hypothesis 5. As an ascending version of Vickrey, iBEA generates significantly high profits for the human bidder. We find it interesting that a high quiz score at the end of the instruction significantly increases bidder profit, indicating that those who have better understanding of the rules of the auction end up doing better. Part of this might be due to the fact that they might be better able to imitate the sophisticated robot strategies.

Result 6 (Revenue). Vickrey auctions generate significantly higher revenue than iBEA. The number of random robots significantly decreases revenue.

Support. In Table 10, for specification (3), the coefficient for Mechanism is positive and significant. The coefficient for the number of random robots is negative and significant. ∎

By Result 6, we reject Hypothesis 6. Unlike the theoretical prediction of equal performance in terms of revenue, the sealed-bid Vickrey auctions generate significantly higher revenue than the ascending *i*BEA. This result is particularly interesting in light of the concern in the literature that Vickrey auctions might generate low revenue. This result could be due to the fact that each human bidder competes against two robots in our experimental setting, which makes it impossible to collude. This points to the obvious next step in research, which is to implement the Vickrey package auctions among human bidders and check if this result still holds. The next measure, efficiency, take into both bidder profit and seller revenue into consideration.

Result 7 (Efficiency). Vickrey generates significantly higher efficiency than *i*BEA. The number of random robots significantly decreases efficiency, while a higher quiz score significantly improves efficiency.

Support. In Table 10, for specification (4), the coefficient for Mechanism is positive and significant. The coefficient for the number of random robots is negative and significant, while that for the quiz score is positive and significant. ∎

By Result 7, we reject Hypothesis 7. Empirical evidence from past laboratory studies of Vickrey and its ascending bid equivalence shows that, in the single unit case and the multi-unit homogeneous object case, the ascending auction usually achieves higher efficiency (Kagel 1995). This difference in performance is usually attributed to feedbacks in the ascending bid auction which makes it more transparent (Kagel, Kinross and Levin 2003). Our results show that with package bidding, Vickrey generates significantly higher efficiency than the ascending *i*BEA.

6 Conclusion

The increasingly popular applications of package auctions to procurement and complex resource allocation problems have stimulated a large body of research in the theory of combinatorial auctions in economics and computer science. Because of the concerns over the deficiencies of Vickrey auctions, several new ascending package auctions have been proposed to implement efficient allocations under various assumptions. One of the most prominent new ascending package auctions is *i*BEA, which achieves approximately efficient allocation and implements Vickrey payments under minimal assumptions on preferences.

As the first step to make a new institution operational and put it to use as an actual economic process that solves naturally occurring problems, we observe the performance of *i*BEA in the context of the simple situations that can be created in a laboratory and assess its performance relative to a natural and important benchmark, the sealed-bid Vickrey auction.

As the first experimental study of *i*BEA in comparison with Vickrey, we use a simple environment where each human bidder competes against two robots with different levels of intelligence. This implementation creates an environment free from the strategic uncertainties inherent in interactions between human bidders.

We have several surprising findings. First, unlike the single-unit Vickrey where bidders tend to overbid in the laboratory, most of our bidders either underbid or bid their true value. A payoff-assessment learning model captures the learning dynamics in this complicated auction better than the reinforcement learning model, indicating that myopic hill-climbing is a reasonable characterization of the adjustment dynamics. Second, in terms of aggregate performance, while bidder profit is significantly higher under *i*BEA, Vickrey generates significantly higher revenue and efficiency than *i*BEA. This result is particularly interesting in light of the general concern in the literature (Milgrom 2004) that package Vickrey auctions might generate low revenue. Admittedly, our result on revenue might be a consequence of our experimental setting where a human bidder competes with two robots, making it impossible to collude. Nonetheless, it has important implications with the increasingly popular use of automated agents. It also points to the natural next step of the research, which is to let human bidders compete against other humans to check if they are able to collude to lower revenue. Lastly, when human bidders compete against robots in a complex environment such as combinatorial auctions, they learn from their robot opponents when the robot strategies are intelligent (e.g., myopic best responses).

As the first laboratory study of *i*BEA, we raise several issues regarding the new mechanism, which warrants further empirical study. The first issue is the tradeoff between the speed of the auction and efficiency of the auction by setting different price increment, ϵ. The larger the auctioneer sets the price increment, the faster the auction converges, however, the final allocation might be further away from the efficient allocation. In an auction with a large number of bidders, we expect the second phase of *i*BEA to take longer, thus it is important to quantify the efficiency-speed tradeoff. The second issue is whether bidders can detect the second phase of the auction and thus collude to increase bidder profit. Like Vickrey, the use of robots in our experiment makes second phase collusion impossible. Therefore, one should interpret our results as empirical evaluations in the absence of collusion.

Empirical investigations of package auctions have important implications not only in procurement and privatization, but also potentially in the allocation of scarce equipment time in large scientific collaboratories.[14] The National Science Foundation and other federal agencies are poised to make significant investments in expanding the ability of geographically distributed groups of scientists to conduct research via the Internet. There are currently over eighty collaboratories in use within multiple scientific communities, including space physics, HIV/AIDS, software engineering, and neuroimaging (Finholt 2002). In many collaboratories, a critical feature of the equipment time allocation problem is that contiguous time slots are more valuable than the sum of separate slots, i.e., user valuation for multiple slots exhibits synergy. Therefore, package auctions might be an important mechanism in achieving efficient allocation of equipment time. This potential application, in fact, motivates the current study, as well as our next research project, where human bidders competing against each other for equipment time in a setting modelled after the earthquake engineering collaboratory.

[14]First proposed in the late eighties, a collaboratory is a center without walls, in which researchers can perform their research without regard to physical location - interacting with colleagues, accessing instrumentation, sharing data and computational resources, and accessing information in digital libraries (Wulf 1993).

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		bidder 1			bidder 2			bidder 3		
Package		A	B	AB	A	B	AB	A	B	AB
Value		10	0	10	0	30	30	4	14	22
Phase	Round	(Bid, Price)			(Bid, Price)			(Bid, Price)		
I	1	**(A, 0)**		(AB, 0)		**(B, 0)**	(AB, 0)			(AB, 0)
I	2	(A, 0)		(AB, 0)		(B, 0)	(AB, 0)		(B, 0)	**(AB, 5)**
I	3	**(A, 5)**		(AB, 5)		**(B, 5)**	(AB, 5)		(B, 0)	(AB, 5)
I	4	**(A, 5)**		(AB, 5)		**(B, 5)**	(AB, 5)		(B, 5)	(AB,10)
I	5	**(A, 5)**		(AB, 5)		(B, 5)	(AB, 5)	(A, 0)	**(B,10)**	(AB,15)
I	6	**(A, 5)**		(AB, 5)		(B,10)	(AB,10)	(A, 0)	**(B,10)**	(AB,15)
I	7	**(A, 5)**		(AB, 5)		**(B,15)**	(AB,15)	(A, 0)	(B,10)	(AB,15)
I	8	**(A, 5)**		(AB, 5)		**(B,15)**	(AB,15)	*(A, 0)*	*(B,10)*	(AB,20)
I	9	**(A, 5)**		(AB, 5)		**(B,15)**	(AB,15)	*(A, 0)*	*(B,10)*	*(AB,20)*
II		(A, 5)		(AB, 5)		~~**(B,15)**~~	~~(AB,15)~~	*(A, 0)*	*(B,10)*	***(AB,20)***
II	10	**(A,10)**		(AB,10)		~~**(B,15)**~~	~~(AB,15)~~	*(A, 0)*	***(B,10)***	*(AB,20)*
II		~~**(A,10)**~~		~~(AB,10)~~		(B,15)	(AB,15)	*(A, 0)*	*(B,10)*	***(AB,20)***
II	11	~~**(A,10)**~~		~~(AB,10)~~		**(B,20)**	(AB,20)	***(A, 0)***	*(B,10)*	*(AB,20)*

Notes:
1. Boldface indicates the winning bid after the current round.
2. Italics indicates bids with the Last-and-Final option.
3. ~~(Bid,Price)~~ indicates that the (Bid, Price) pair is excluded.

iBEA	Package	Price	Piv. Revenue	Rebate	Final Price	Profit
bidder 1	A	10	20	10	0	10
bidder 2	B	20	20	10	10	20
bidder 3	-	-	-	-		0
auctioneer	-	-	-	-		10
Vickrey	Package	Price	Piv. Revenue	Rebate	Final Price	Profit
bidder 1	A	10	34	6	4	6
bidder 2	B	30	24	16	14	16
bidder 3	-	-	-	-		0
auctioneer	-	-	-	-		18

Note: Piv. Revenue refers to total revenue when bidder i is excluded.

Table 1: A Simple Example of iBEA

Mechanism	Robot Strategies	Information	Notation	# of Subjects	Exchange Rates
		High	iSS_h	10	2
*i*BEA	Sincere, Sincere	Low	iSS_ℓ	10	2
		High	iSR_h	9	1.5
*i*BEA	Sincere, Random	Low	iSR_ℓ	8	1.5
		High	iRR_h	8	1
*i*BEA	Random, Random	Low	iRR_ℓ	10	1
		High	vSS_h	10	1.25
Vickrey	Sincere, Sincere	Low	vSS_ℓ	10	1.25
		High	vSR_h	10	1.25
Vickrey	Sincere, Random	Low	vSR_ℓ	10	1.25
		High	vRR_h	10	1.25
Vickrey	Random, Random	Low	vRR_ℓ	10	1.25

Table 2: Features of Experimental Sessions

Package	Active Bids	Bid/Value	Underbidding	Truthful Bidding	Overbidding
A	0.700	0.989	0.538	0.271	0.190
B	0.663	0.995	0.503	0.307	0.191
C	0.668	1.184	0.544	0.284	0.172
D	0.653	0.875	0.531	0.296	0.173
AB	0.860	1.029	0.572	0.217	0.211
AC	0.708	0.964	0.607	0.224	0.169
AD	0.702	0.908	0.620	0.219	0.162
BC	0.713	0.959	0.605	0.206	0.189
BD	0.697	0.948	0.610	0.208	0.182
CD	0.730	0.917	0.584	0.249	0.167
ABC	0.835	0.978	0.605	0.202	0.194
ABD	0.802	0.953	0.613	0.200	0.187
ACD	0.743	0.954	0.596	0.231	0.173
BCD	0.722	0.962	0.580	0.238	0.182
ABCD	0.827	1.005	0.597	0.198	0.206

Table 3: Proportion of Active Bids, Bid/Value Ratio and Proportion of Under-, Truthful, and Over-Bidding in Vickrey

Dependent Variable: Active Bids		
	(1)	(2)
Value	0.008	0.008
	(0.002)***	(0.003)***
D_{AB}	0.085	0.086
	(0.021)***	(0.022)***
D_{HS}	0.164	
	(0.072)**	
D_{info}		-0.017
		(0.077)
Observations	9000	9000

Notes:
1. Coefficients are probability derivatives.
2. Robust standard errors in parentheses are adjusted for clustering at the individual level.
3. Significant at: ** 5% level; *** 1% level.

Table 4: Probit: Factors Affecting the Likelihood of Active Bids in Vickrey

	Dependent Variable: Bid	
	(1)	(2)
Value	0.962	0.937
	(0.029)***	(0.028)***
Auction		0.045
		(0.115)
D_{HS}		-0.874
		(0.634)
Cum. Profit		-0.053
		(0.042)
D_{AB}		0.526
		(0.327)
Constant		0.875
		(0.789)
Observations	6614	6614
R-squared	0.81	0.52

Notes:
1. Robust standard errors in parentheses are adjusted for clustering at the individual level.
2. Significant at: *** 1% level.

Table 5: OLS: Bidding Decision in Vickrey

	Dependent Variable: PlaceBid					
	(1)	(2)	(3)	(4)	(5)	(6)
	SS	SS	SR	SR	RR	RR
Temp. Profit	0.013	0.010	0.016	0.014	0.013	0.013
	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***
D_{AB}	0.045	0.020	0.031	0.020	0.050	0.046
	(0.015)***	(0.013)	(0.015)**	(0.014)	(0.015)***	(0.016)***
D_{info}	0.078	-0.025	0.095	0.019	0.074	0.054
	(0.057)	(0.048)	(0.053)*	(0.058)	(0.038)**	(0.050)
D_{info}*MBR		0.271		0.144		0.041
		(0.096)***		(0.027)***		(0.034)
Observations	23643	23643	16506	16506	13680	13680

Notes:

1. Coefficients are probability derivatives.
2. Robust standard errors in parentheses are adjusted for clustering at the individual level.
3. Significant at: * 10% level; ** 5% level; *** 1% level.

Table 6: Probit: Likelihood of MBR Bidding in *i*BEA

Treatment	Mean Temp Profit	Negative	Non-Negative	Total	Correct Rate
SS_ℓ	3.97	38	79	117	32.48%
SR_ℓ	3.86	5	80	85	5.88%
RR_ℓ	10.16	0	19	19	0.00%
SS_h	1.36	128	108	236	54.24%
SR_h	-0.25	97	107	204	47.55%
RR_h	4.56	2	16	18	11.11%

Table 7: Last-and-Final in *i*BEA

Dependent Variable: Temp. Profit of Last-and-Final Bids		
	(1)	(2)
Round	-0.390	-0.399
	(0.101)***	(0.102)***
D_{info}	-3.786	-3.061
	(1.367)***	(1.586)*
D_{HS}		-1.501
		(1.072)
Constant	6.943	7.001
	(1.525)***	(1.525)***
Observations	679	679
R-squared	0.18	0.19

Notes:
1. Robust standard errors in parentheses are adjusted for clustering at the individual level.
2. * significant at: * 10% level; ** 5% level; *** 1% level

Table 8: OLS: Temporary Profit of Last-and-Final Bids

	Human Profit			Total Profit		
Treatment	*i*BEA	Vickrey	Vickrey (grid 5)	*i*BEA	Vickrey	Vickrey (grid 5)
SS_ℓ	3.04	2.72	2.23	16.29	10.85	8.75
SR_ℓ	3.56	1.80	2.46	18.15	8.56	8.70
RR_ℓ	7.02	3.30	2.96	18.27	9.58	9.33
SS_h	2.71	2.48	2.14	16.84	10.06	9.18
SR_h	3.75	2.96	2.89	17.77	9.75	8.99
RR_h	7.05	2.73	2.67	17.85	10.10	9.53
	Revenue			Efficiency		
robots	*i*BEA	Vickrey	Vickrey (grid 5)	*i*BEA	Vickrey	Vickrey (grid 5)
SS_ℓ	13.87	20.12	20.90	92.2%	98.4%	94.02%
SR_ℓ	8.86	20.04	19.70	84.8%	90.3%	89.60%
RR_ℓ	7.13	18.88	18.75	78.3%	89.7%	88.28%
SS_h	13.05	20.44	20.45	93.0%	97.4%	94.49%
SR_h	9.60	19.92	20.20	87.3%	93.0%	91.40%
RR_h	5.82	17.81	17.80	73.2%	88.0%	86.34%

Table 9: Aggregate Performance of the Two Mechanisms

	(1)	(2)	(3)	(4)
Dependent Variable:	Human Profit	Total Profit	Revenue	Efficiency
Mechanism	-2.422	-8.389	9.386	0.043
	(0.460)***	(0.503)***	(0.487)***	(0.010)***
D_{info}	0.002	0.291	-0.351	0.003
	(0.351)	(0.431)	(0.406)	(0.008)
# of Random Robots	1.150	0.505	-2.354	-0.060
	(0.212)***	(0.258)*	(0.277)***	(0.005)***
Quiz Score	0.816	-0.223	0.850	0.023
	(0.378)**	(0.459)	(0.436)*	(0.010)**
Constant	2.246	26.225	-0.746	0.764
	(1.352)*	(1.822)***	(1.634)	(0.041)***
Observations	1150	1150	1150	1150
R-squared	0.09	0.28	0.42	0.19

Notes:

1. Robust standard errors in parentheses are adjusted for clustering at the individual level.
2. Significant at: * 10%; ** 5%; *** 1% level.

Table 10: OLS: Factors Affecting Aggregate Performance of the Mechanisms

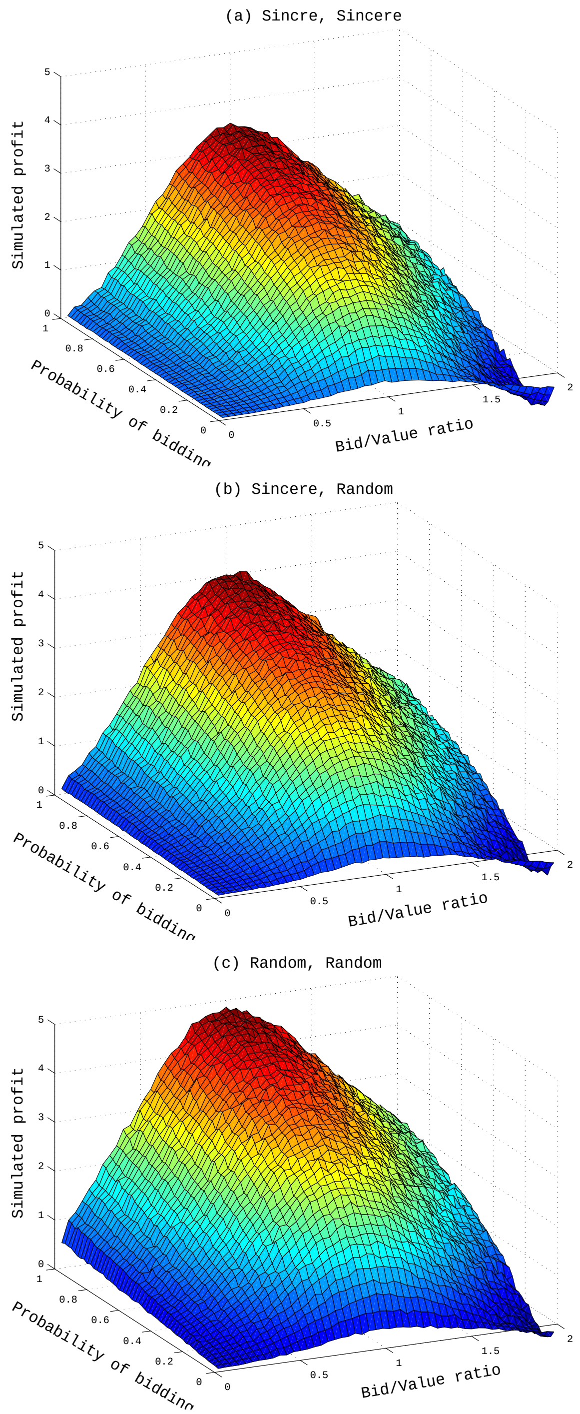

Figure 1: Simulated Profit for Human Bidder under Three Environments in Vickrey

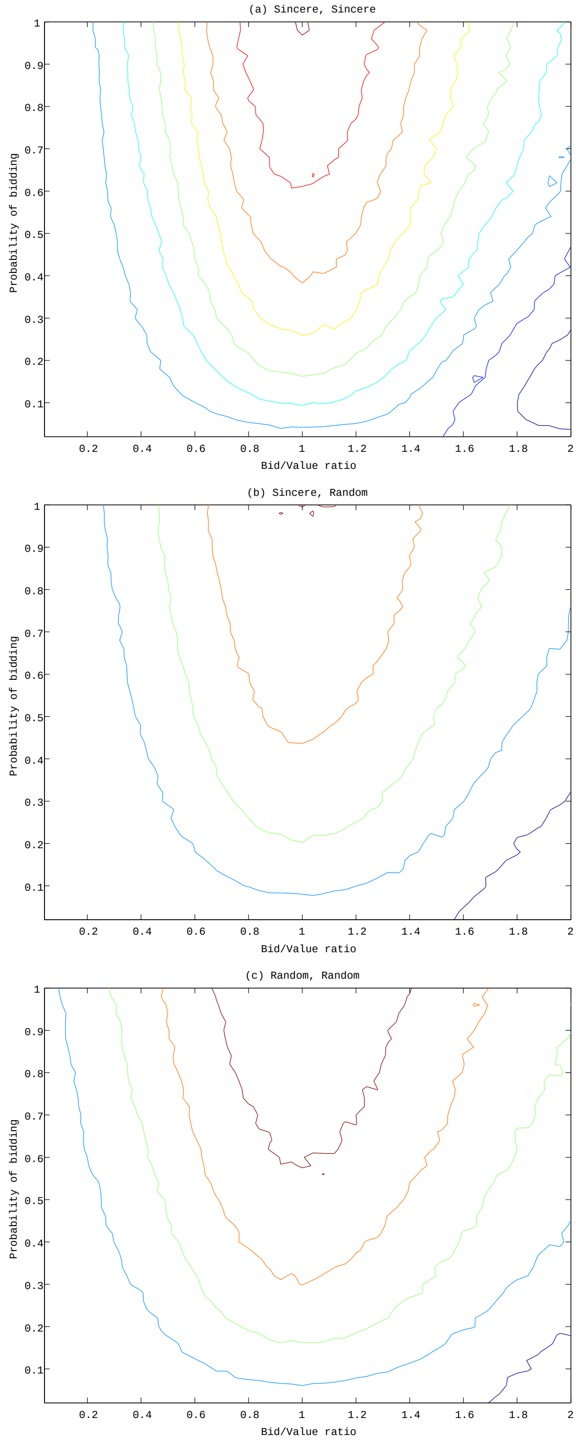

Figure 2: Contour of Simulated Profit for Human Bidder under SS Environment in Vickrey

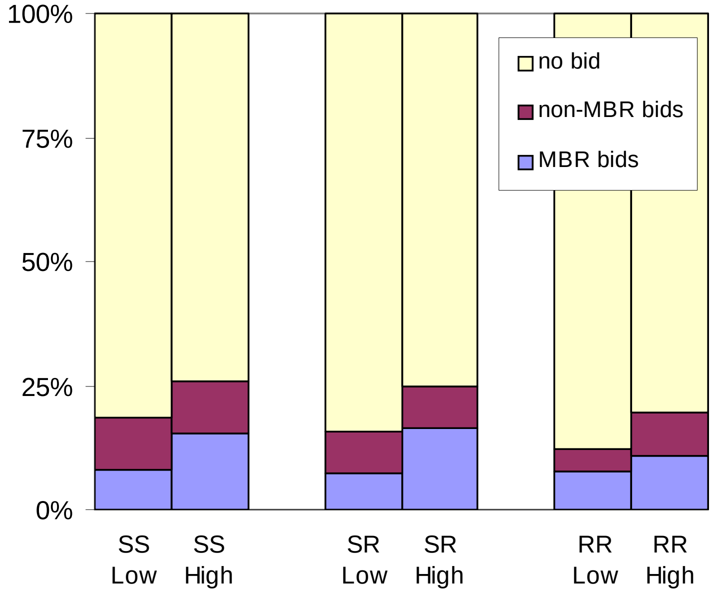

Figure 3: Proportion of MBR and non-MBR Bids in *i*BEA

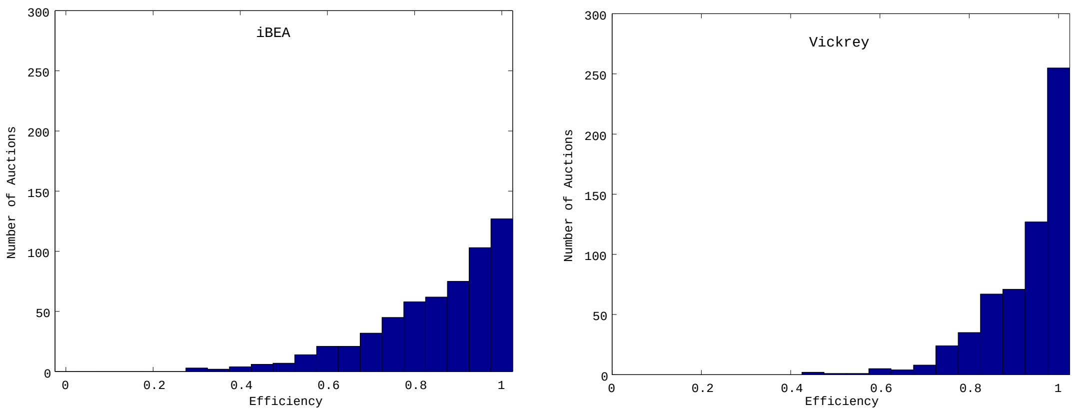

Figure 4: Distribution of Observed Efficiency