An Experimental Study of Serial and Average Cost Pricing Mechanisms*

Yan Chen

Department of Economics, University of Michigan, Ann Arbor, MI 48109-1220 Phone:(734)763-9619; Fax: (734)764-2769; Email: yanchen@umich.edu

Revised: August 2001

Abstract

This paper reports the first experimental study of the serial and the average cost pricing mechanisms under five different treatments: a complete information treatment and four treatments designed to simulate distributed systems with extremely limited information, synchronous and asynchronous moves. Although the proportion of Nash equilibrium play under both mechanisms is statistically indistinguishable under complete information, the serial mechanism performs robustly better than the average cost pricing mechanism in distributed systems, both in terms of the proportion of equilibrium play and system efficiency.

Keywords: serial mechanism, asynchrony, experiment

JEL Classification: C91, D83

^{*}I am indebted to Scott Shenker for his many helpful comments and suggestions. I thank Julie Cullen, Dan Friedman, Eric Friedman, Drew Fudenburg, Amy Greenwald, Teck-Hua Ho, Lutz Kilian, Bettina Kuon, Wolfgang Lorenzon, David Lucking-Reiley, Richard McKelvey, Rosemarie Nagel, Tom Palfrey, David Pennock, Charles Plott, Rajiv Sarin, Katerina Sherstyuk, Alison Watts, and seminar participants at Michigan, Mississippi, Rutgers, Texas A&M, the 1998 Econometric Society Winter meetings (Chicago), ESA 1998 (Mannheim, Tucson) for discussions and comments, Yuri Khoroshilov for excellent research assistance, Bruce Bell for programming the experiment and Wirtschaftspolitische Abteilung at the University of Bonn for their hospitality. The research support provided by the NSF grant SBR-9805586, SES-0079001, a grant from the University of Michigan and the Deutsche Forschungsgemeinschaft through SFB303 at the Universität Bonn, the support of the EEPS Laboratory at Caltech, and the Dean's office at the Pasadena City College are gratefully acknowledged. I am grateful to two anonymous referees for their constructive comments which significantly improved the paper. Any remaining errors are my own.

1 Introduction

In a wide variety of real world situations a group of agents share a common production process transforming input into output. Examples of shared resources include computing facilities, secretarial support and lab facilities within an organization. A cost-sharing mechanism distributes the service and allocates the corresponding costs to each agent. Two prominent cost-sharing mechanisms are the serial mechanism (Shenker (1990), Moulin and Shenker (1992)) and the average cost pricing mechanism (see Tauman (1988) for a survey). We will use two examples to illustrate how the mechanisms work.¹

We first look at a group of ranchers who might share the cost of constructing and maintaining an irrigation network. Aadland and Kolpin (1998) provide an empirical and axiomatic analysis of cost-sharing arrangements of irrigation ditches located in south-central Montana. In their sample, a typical ditch begins at the headgate and then continues on a sequential path through the lands of each rancher using the main ditch. Ranchers' private ditches branch off from the main ditch and transport water to their land. The costs associated with the main ditch are shared among the ranchers. Kolpin and Aadland (2001) find that the cost sharing rules employed on these ditches are variations of the average and serial cost sharing mechanisms. A rule is in the average class if all agents pay according to an identical fixed "rate", which may be defined on a per capita basis, per irrigated acre basis, etc. A serial rule partitions the ditch into "a sequence of segments such that all agents require the first segment to be operational in order to receive water, all but the first agent on the ditch additionally require the second segment to be operational, Each segment is then treated like a separate ditch whose costs are covered by having all agents requiring its use pay an identical fixed rate." (Kolpin and Aadland (2001)) An agent's total cost share is the sum of his obligations on each of these individual segments. This example provides a more traditional setting where the ranchers know the rules of the game as well as each other's demand fairly well.

A more recent example is provided by the Internet, which has becoming increasingly important in global telecommunications. In the context of several agents sharing a network link, the cost to be shared is congestion experienced. Each agent controls the rate at which she is transmitting data. If the sum of the transmission rates is greater than the total link capacity, then the link becomes congested and the agents' packets experience delays. Most current Internet routers use a FIFO packet scheduling algorithm, which results in each agent's average queue proportional to her transmission rate. This corresponds to the average cost pricing mechanism (Shenker (1990)). In contrast,

¹A formal definition of each mechanism is provided in Section 2.

the Fair Queueing packet scheduling algorithm, which corresponds to the serial mechanism, leads to congestion allocations such that an agent's average queue is independent of transmission rates higher than her own. The latter has been proposed as an alternative to the former, based on theoretical and simulation results (Stoica, Shenker and Zhang (1998)). The new generation of Cisco 7200, 3600 and 2600 rounters have both the FIFO and Fair Queueing options.² This paper presents the first experimental study of the performance of the two mechanism in both the traditional complete information setting and the Internet setting.

Most of the theoretical literature has focused on the axiomatic characterization of these mechanisms (e.g., Moulin and Shenker (1994), Friedman and Moulin (1999)) and their static properties in a complete information setting with synchronous actions. However, as Friedman and Shenker (1998) pointed out, in a *distributed system*³ such as the Internet where agents have very limited *a priori* information about other agents and the payoff structure and where there is no synchronization of actions, traditional solution concepts that we use to characterize these mechanisms, such as Nash equilibrium or even the serially undominated set⁴, might not be achieved as a result of learning. They propose new solution concepts for distributed systems describing convergence for learning algorithms satisfying certain theoretical properties.

Although Friedman and Shenker (1998) used the Internet as the context for their new theory, limited information and asynchrony are more realistic assumptions than complete information and synchronous play in many real economic situations. To my best knowledge there have not been experimental studies that incorporate both limited information and asynchrony to study implementation. The average cost pricing mechanism has not been studied in the laboratory either. The only other experimental study of the serial mechanism is by Dorsey and Razzolini (1999). They investigate the performance of the serial mechanism with each human subject against three computerized players, where each human player knows his own cost share and payoff structure but has no information about the opponents' payoff structures. Their information condition is in between the complete information and limited information setting in this study. They do not consider the performance of the mechanism under limited information or asynchrony.

In this paper I design an experiment to evaluate the serial and the average cost pricing mechanism in an easy environment with complete information, and more challenging environments with

²http://www.cisco.com/warp/public/121/7200_per-vc-CBWFQ.htm

³Following Friedman and Shenker (1998), a system is called a *distributed system* "because the users are geographically dispersed and are accessing the resource through the network." The Internet is a prominet example.

⁴Serially undominated set is the set of outcomes of a strategic game that survives iterated elimination of strictly dominated actions.

extremely limited information and asynchronous moves. The goal of this paper is to compare the performance of the two mechanisms in various settings and to assess the plausibility of the new solution concepts.

The paper is organized as follows. Section 2 introduces the theoretical properties of the serial (sometimes shortened as SER) and average cost pricing (hereafter shortened as ACP) mechanisms. Section 3 presents the experimental design. Section 4 compares the performance of the mechanisms under complete information. Section 5 presents results in distributed systems with limited information and asychrony. Section 6 discusses the robustness of the experimental results with respect to changes in the environment. Section 7 concludes the paper.

2 The Serial and ACP Mechanisms - Theoretical Properties

Suppose a group of n agents share a one-input, one-output technology with decreasing returns. Each of the n agents announces his demand q_i of output. Each agent gets her demand q_i and pays a cost share, x_i . Note x_i is the total cost agent i pays. In the irrigation example, q_i corresponds to the total amount of maintenance of the main ditch demanded by agent i, while x_i is what agent i pays to get the maintenance done. In the example of Internet routers, q_i is agent i's data transmission rate, while x_i is the congestion, i.e., the average queue experienced by agent i. Therefore, x_i is the reduction in agent i's utility due to congestion. Let $q_1 \leq q_2 \leq \cdots \leq q_n$. The cost function is denoted by C, which is strictly convex. A cost-sharing mechanism must allocate the total cost $C(\sum_i q_i)$ among the n agents.

The serial mechanism, originally introduced by Shenker (1990), was analyzed by Moulin and Shenker (1992) in the context of cost and surplus sharing with complete information. The mechanism can be characterized by four properties: unique Nash equilibrium at all profiles⁵, anonymity (the name of the agents does not matter), monotonicity (an agent's cost share increases when she demands more output) and smoothness (an agent's cost share is a continuously differentiable function of the vector of demands). Among agents endowed with convex, continuous and monotonic preferences, the serial mechanism is the only cost sharing rule which is dominance-solvable and its unique Nash equilibrium is also robust to coalitional deviations when agents cannot transfer outputs.

Under the serial mechanism, agent 1 (with the lowest demand) pays (1/n)th of the cost of producing nq_1 . Agent 2 pays agent 1's cost share plus 1/(n-1)th of the incremental cost from

⁵Assume agents have convex, continuous and monotonic preferences.

 nq_1 to $(n-1)q_2 + q_1$, i.e.,

$$x_1^S(c,q_1) = C(nq_1)/n; \text{ and } x_2^S(c,q_1,q_2) = \frac{C(nq_1)}{n} + \frac{C(q_1 + (n-1)q_2) - C(nq_1)}{n-1}.$$

And so on. Therefore, an agent's cost share under the serial mechanism is only affected by her own demand and those whose demands are lower than hers. That is, an agent's cost share is independent of demands higher than her own.

Like the serial mechanism, the average cost pricing mechanism satisfies anonymity, monotonicity and smoothness. It is the only method that is robust to arbitrage, i.e., agents cannot benefit from merging or splitting their demands. In contrast to the serial mechanism, the normal form game induced by the average cost pricing mechanism is in general not dominance-solvable, nor does it have a unique equilibrium at all profiles when agents have convex, continuous and monotonic preferences.

When agent *i* demands q_i amount of output, the general formula for agent *i*'s cost share under the average cost pricing mechanism is given by

$$x_i^A(c,q) = (q_i / \sum_k q_k) \cdot C(\sum_k q_k), \text{ for all } i = 1, \cdots, n.$$

Therefore, under ACP an agent's cost share is proportional to her demand. It is affected by her own demand, and the sum of all other agents' demands.

There is no systematic efficiency comparison between the two mechanisms. In general there exists no differentiable and monotonic cost sharing mechanism where all Nash equilibrium outcomes are first best Pareto optimal at all preference profiles. Moulin and Shenker (1992) provide a definition of second best efficiency⁶ and show that the serial mechanism yields a second best efficient equilibrium while ACP does not.

A particularly interesting question is the performance of the two mechanisms in distributed systems where users are geographically dispersed and are accessing the resource through the network. Friedman and Shenker (1998) address the issue of learning and implementation in distributed systems. They argue that when agents have very limited *a priori* information about the other players

⁶"For an arbitrary cost sharing mechanism ξ , say that (q_1, \dots, q_n) is a Nash equilibrium outcome at some utility profile. We ask if there is another vector of demands (q'_1, \dots, q'_n) such that at the corresponding allocation dictated by the mechanism ξ , no one is worse off and someone is better off than at the equilibrium allocation corresponding to (q_1, \dots, q_n) . If no such vector of demands exists, we call our equilibrium second best efficient." Moulin and Shenker (1992, p.1025)

and the payoff structure, standard solution concepts like Nash equilibrium or even the serially undominated set are not necessarily achieved as a result of learning in the network setting. Therefore, new solution concepts, such as the serially unoverwhelmed set and the Stackelberg undominated set are proposed. Loosely speaking, one action *overwhelms* another if all payoffs, over all sets of other players' actions, for the one are greater than all payoffs, over all sets of other players' actions, for the other⁷. Therefore, if action U overwhelms action D, then U dominates D, but the converse is not true.

		Play	Player 2								
		L	R								
Player 1	U	$\pi_1(UL), \pi_2(UL)$	$\pi_1(UR), \pi_2(UR)$								
	D	$\pi_1(DL), \pi_2(DL)$	$\pi_1(DR), \pi_2(DR)$								

For example, in the above 2×2 game, action U dominates D if $\pi_1(UL) \ge \pi_1(DL)$ and $\pi_1(UR) \ge \pi_1(DR)$; action U overwhelms D if $\min{\{\pi_1(UL), \pi_1(UR)\}} \ge \max{\{\pi_1(DL), \pi_1(DR)\}}$. The *seri*ally unoverwhelmed set is the set remaining after iterated elimination of overwhelmed actions. One main result of Friedman and Shenker (1998) is that reasonable learners⁸ converge to the serially unoverwhelmed set. In comparison, Milgrom and Roberts (1990) showed that adaptive learners converge to the serially undominated set. A game is *D*-solvable if iterated elimination of dominated strategies leads to a single eventual outcome. A game is *O*-solvable if iterated elimination of overwhelmed strategies leads to a single eventual outcome. Among the cost sharing mechanisms, the serial mechanism is O-solvable⁹ while ACP is not.

3 Experimental Design

The experimental design reflects both theoretical and technical considerations. The goal of the design is to compare the performance of the serial and ACP mechanisms in two different settings: a complete information setting that tests the prediction of dominance-solvability, and a more challenging network setting to compare the performance of the two mechanisms and to assess the plausibility of the new solution concepts. The economic environment and experimental procedures are discussed in the sections below.

⁷See Friedman and Shenker (1998) for a precise definition.

⁸The key components of a reasonable learner are optimization, monotonicity and responsiveness. See Friedman and Shenker (1998).

⁹This is proved in Theorem 8 in Friedman and Shenker (1998).

3.1 The Economic Environment

In a simple environment to test the serial and ACP mechanism under various treatments, agents are endowed with linear preferences $\pi_i(x_i, q) = \alpha_i q_i - x_i$, where α_i is agent *i*'s marginal utility for the output, and x_i is her cost share. The cost function is chosen to be quadratic, $C(q) = q^2$. In the network context with several agents sharing a network link, α_i is agent *i*'s value for the amount of data transmitted per unit of time, and the cost to be allocated corresponds to the congestion experienced. Therefore, the cost should be interpreted as the reduction in agent *i*'s utility due to congestion. I chose linear utility and quadratic cost functions in order to get a unique interior Nash equilibrium. In Section 6 I present simulation results for more general utility and cost functions.

Consider a two-player game with $\alpha_1 \leq \alpha_2$. Then under the serial mechanism, the cost share for agent 1 is $x_1^S = C(2q_1)/2 = 2q_1^2$. Agent 2 picks up the remaining cost, $x_2^S = C(q_1 + q_2) - C(2q_1)/2 = (q_1 + q_2)^2 - 2q_1^2$. The unique, dominance-solvable Nash equilibrium is thus characterized by

$$q_1^S = \frac{\alpha_1}{4}$$
, and $q_2^S = \frac{1}{4}(2\alpha_2 - \alpha_1)$.

The Stackelberg equilibrium for the serial game coincides with the above Nash equilibrium¹⁰.

For the ACP mechanism, the cost shares of the two agents are $x_1^A = \frac{q_1}{q_1+q_2} \cdot C(q_1 + q_2) = q_1(q_1 + q_2)$, and $x_2^A = \frac{q_2}{q_1+q_2} \cdot C(q_1 + q_2) = q_2(q_1 + q_2)$ respectively. Therefore, the unique dominance-solvable Nash equilibrium is characterized by

$$q_1^{A_n} = \frac{2\alpha_1 - \alpha_2}{3}$$
, and $q_2^{A_n} = \frac{2\alpha_2 - \alpha_1}{3}$.

Note that the Stackelberg equilibria under ACP usually differ from the Nash equilibrium. The Stackelberg equilibrium with player 2 as the leader is

$$q_1^{A_{s2}} = \frac{3\alpha_1}{4} - \frac{\alpha_2}{2}$$
, and $q_2^{A_{s2}} = \alpha_2 - \frac{\alpha_1}{2}$.

In the asynchronous treatment discussed in Section 3.2 player 2 will be the Stackelberg leader.

The mechanisms are implemented as normal form games with a discrete strategy space, $S_i = \{1, 2, \dots, 11, 12\}$ for each *i*. Parameters are chosen to ensure: (1) the serial game is both D-solvable and O-solvable, while the ACP game is D-solvable but not O-solvable; (2) the Stackelberg equilibrium and Nash equilibrium under ACP are sufficiently far away from each other; (3) the

¹⁰This is a general property of the SER mechanism (Corollary 1 to Theorem 1 in Moulin and Shenker (1992)).

normal form games with a discrete strategy space has a unique Nash equilibrium under ACP¹¹; (4) most of the payoffs are positive in both normal form games. Note that since the stage game under SER and ACP are both dominance-solvable, the equilibrium of the complete information repeated game is simply repeated play of the static equilibrium.

[Table 1 about here.]

Table 1 reports the parameters and equilibrium quantities for each type of equilibrium for the two mechanisms. In the second column we let $\alpha_1 = 16.1$ and $\alpha_2 = 20.1$. Under the serial mechanism the Nash as well as the Stackelberg equilibrium quantities are (4, 6). Under ACP, the Nash equilibrium quantities are (4, 8), while the Stackelberg equilibrium quantities with player 2 as the leader are (2, 12). Note that we use Blue for player 1 and Red for player 2 in the instructions (see Appendix).

3.2 Experimental Procedures

I implement five different treatments. For a baseline treatment I conducted twelve sessions of the serial and ACP mechanisms under complete information with the round robin design (hereafter shortened as **Round Robin**). Each session has eight pairs of players. Each of the player 1's is matched with each of the player 2's only once. The entire session lasts for eight rounds. Under the Round Robin treatment, each player is given complete information about the payoff matrix and the structure of the game. They are also given information about quantities chosen and the corresponding payoffs of all players. This treatment is designed to compare the performance of the two mechanisms as one-shot games under complete information. The natural solution concept for this treatment is dominance-solvability.

To evaluate the possibility of applying these mechanisms to distributed systems such as the Internet, I designed four treatments with limited information and various degrees of asynchrony. Learning in distributed systems is characterized by two features. First, players have extremely limited information - they often do not know the payoff functions, nor do they know how their payoffs depend on the actions of others, probably due to the lack of information about the detailed nature of the resources itself. Therefore, in the experimental setup the only information players

¹¹I thank Scott Shenker for suggesting using non-integer $\{\alpha_i\}_i$ to avoid the multiple equilibria problem in ACP. In environments with linear preferences and quadratic cost functions even though there exists a unique Nash equilibrium in a continuous strategy space under the ACP mechanism, there are multiple equilibria when the strategy space is discrete and the preference parameters, $\{\alpha_i\}_i$, are integers. The proof is available from the author upon request.

have is their own action and the resulting own payoffs. Second, there is no synchronization. The rate at which updating occurs can vary by many orders of magnitude. This feature is reflected by the following design: two treatments with synchronous play and updating, two with asynchronous play and updating. In the synchronous treatments (hereafter shortened as **SYN**) every player receives his own payoff feedback after each round. In the asynchronous treatments (hereafter shortened as **ASYN**) player 1 submits a demand and gets a payoff feedback after each round, but player 2 submits a demand which is matched with his opponents' demands for the next five rounds and gets a cumulative payoff feedback every five rounds. Therefore, in the asynchronous treatment player 2 acts five times slower than player 1 and becomes the *de facto* Stackelberg leader¹².

With both synchronous and asynchronous play, I designed one treatment where players are randomly re-matched into pairs in each of the 150 rounds, and another treatment where players are matched into fixed pairs at the beginning of each session, and play the same partner for 150 rounds. The former captures the inherent randomness in many network settings, while the latter reflects situations with fixed sets of players, such as cost sharing in irrigation ditches (Aadland and Kolpin (1995)). In all four treatments the game lasts for 150 rounds (30 rounds for player 2 in ASYN) and the players always keep their own type. To summarize, I implement the following five different treatments.

- 1. Round Robin: complete information, round robin;
- 2. SYN_r: limited information, synchronous play, with random re-matching for each of the 150 rounds;
- 3. SYN_f : limited information, synchronous play, with repeated fixed pairs for 150 rounds;
- 4. ASYN_r: limited information, asynchronous play, with random re-matching for each of the 150 rounds; and
- 5. ASYN_f: limited information, asynchronous play, with repeated pairs for 150 rounds.

Computerized experiments were conducted by the author at the EEPS Laboratory at the California Institute of Technology (hereafter shortened as CIT) in June and July, 1997, and the RCGD

¹²Asynchrony as defined by Friedman and Shenker (1998) requires the ratio of expected reaction time of different players to be fixed. Therefore, there can be two different implementations of asynchrony in the experimental setting: a fixed ratio as described above, or more random speed differentials with fixed expected ratio. Both implementation of asynchrony are faithful to the theoretical model. I chose the former because of its simplicity.

Laboratory at the University of Michigan in November, 1999 and January, 2000. Subjects were students and staff from CIT, Pasadena City College (hereafter shortened as PCC) and the University of Michigan (hereafter shortened as UM)¹³. A total of 484 subjects participated in the experiment. No subject was used in more than one session.

[Table 2 about here.]

Table 2 lists the features of each session, including session number and subject pool, number of subjects in each session, mechanisms implemented, and game length under each treatment. At the beginning of each session subjects randomly drew an identification number. Then each of them was seated in front of the corresponding terminal, with a folder containing the instructions and record sheets. After the instructions were read aloud, subjects were required to finish the Review Questions, which were designed to test their understanding of the instructions. Afterwards the experimenter checked answers and answered questions. In all sessions the instruction period was within 20 minutes. There was no practice round in any session. The Round Robin sessions consisted of 8 rounds and typically lasted for 40 minutes. The SYN and ASYN sessions consisted of 150 rounds and typically lasted for one and a half hours. The average earnings of experimental subjects was \$20.16, not including the \$8 participation fee for PCC subjects¹⁴.

Instructions for the experiments are in the Appendix. Note that in both the SYN and ASYN treatments players had extremely limited information - they were told that they were in a game, the game length and their strategy space. At the end of each round each player was informed of his own choice in the previous round and his own payoff corresponding to his previous round's choice of quantity. They had no information about the payoff matrix, nor whom they were playing with.

¹³I checked the subject pool effects by using the data from the fixed-pairs and Round Robin treatments. In the fixed pairs treatments, one-tailed t-tests show that the difference in the proportion of equilibrium play are not significant at the 10% level (z = 2.32 for CIT vs. PCC, z = 1.22 for CIT vs. UM, and z = 1.02 for UM vs. PCC), and that the difference in efficiency are not significant between CIT and UM (z = 1.88), PCC and UM (z = 0.12). The only significant difference in efficiency is CIT > PCC, with z = 2.63 (p < 0.05). Under Round Robin, CIT subjects played Nash equilibrium strategy significantly more than those from PCC (p < 0.01, one-tailed permutation test) while efficiency difference is weakly significant (p = 0.768, one-tailed permutation test).

¹⁴The participation fee was used to compensate the PCC subjects for transportation costs. Since the experiment was conducted on the CIT and UM campuses, subjects from CIT and UM did not receive a participation fee.

4 Performance of the Mechanisms Under Complete Information

In this section I compare the performance of the two mechanisms under the Round Robin treatment, using two criteria - the proportion of Nash equilibrium play and the system efficiency.¹⁵ Under the Round Robin treatment, the theoretical prediction for both mechanisms is the dominance-solvable Nash equilibrium.

[Table 3 about here.]

Table 3 tabulates the proportion of Nash equilibrium play in each round under the Round Robin treatment, as well as the proportion of equilibrium play in all rounds. The last row presents the p-values for one-tailed permutation tests under the null hypothesis that the proportion of Nash equilibrium play is the same under both mechanisms.

RESULT 1 (Equilibrium Play under Round Robin) : In the Round Robin treatment, at the 8th round an average of 88.6% of the subjects played the unique Nash equilibrium strategy under SER; while an average of 80.9% of the subjects played the unique Nash equilibrium strategy under ACP. The proportion of the Nash equilibrium play under the two mechanisms is not significantly different.

SUPPORT. Table 3 presents the proportion of Nash equilibrium play for each round. Permutation tests under the null hypothesis that the proportion of Nash equilibrium play under SER is the same as that under ACP for round t, where $t = 1, 2, \dots, 8$, show that none of the p-values is significant at the five percent level. The overall proportion of Nash equilibrium play under the two mechanisms is not significantly different either (p = .456, one-tailed).

Result 1 is not surprising since both games are dominance-solvable, and the presentation in the form of bimatrix games is fairly transparent. Under complete information we expect that adaptive learning converge to the unique Nash equilibrium.

Although there is no theoretical systematic efficiency comparison between the two mechanisms in general, it is informative to check the actual efficiency of the system in this particular experiment. Group efficiency is calculated by taking the ratio of the sum of the actual earnings of all subjects in a session and the Pareto-optimal earnings of the group. Note that in this experimental setting the Pareto optimal payoff is 970 at strategy two-tuple (1,9) in both SER and ACP. As a

¹⁵A complete set of the data is available from the author upon request.

benchmark, the efficiency of Nash (and Stackelberg) equilibrium under the serial mechanism is 87.63%. Under ACP the efficiency of Nash equilibrium is 83.71%, while the efficiency of the Stackelberg equilibrium with player 2 as leader is 79.71%.

RESULT 2 (Efficiency under Round Robin) : *The efficiency of the serial mechanism is significantly higher than that of the ACP mechanism under the Round Robin treatment.*

[Table 4 about here.]

SUPPORT. The last column of Table 4 shows the efficiency of each session under the Round Robin treatment. Permutation tests show that the efficiency of SER > ACP at a significance level of .023 (one-tailed).

Therefore, under Round Robin although the amount of Nash equilibrium play is not significantly different between the two mechanisms, the serial mechanism generated significantly higher system efficiency than ACP.

5 Performance of the Mechanisms in Distributed Systems

Although the proportion of Nash equilibrium play was not significantly different under the two mechanisms under complete information, the performance of the two mechanisms differed dramatically in distributed systems. In this section I will evaluate the two mechanisms under SYN and ASYN in terms of the proportion of equilibrium play and efficiency, and the plausibility of new solution concepts proposed for distributed systems.

[Table 5 about here.]

Table 5 presents the proportion of Nash and Stackelberg equilibrium play for each independent observation¹⁶ under each of the four different treatments in distributed systems.

RESULT 3 (Equilibrium Play under SER and ACP) : Under all four treatments the ranking of the proportion of equilibrium play is highly significant: SER > ACP.

¹⁶Note under the random matching treatment each session is an independent observation, while under the fixed pair treatment each pair is an independent observation.

SUPPORT. Table 5 presents the proportion of Nash and Stackelberg equilibrium play for each independent observation. One-tailed permutation tests show that the proportion of equilibrium play under SER is greater than the proportion of equilibrium play under ACP, with p = 0.05 under SYN_r, p = 0.05 under ASYN_r, p < 0.01 under SYN_f, and p < 0.01 under ASYN_f.

Therefore, in contrast to Result 1 where the proportion of Nash equilibrium play are not significantly different under the Round Robin treatment, the proportion of Nash and Stackelberg equilibrium play do differ significantly in distributed systems. The SER mechanism induces significantly more equilibrium play than the ACP mechanism.

RESULT 4 (Efficiency under SER and ACP) : Under all four treatments the ranking of group *efficiency is highly significant: SER > ACP.*

[Table 6 about here.]

SUPPORT. Table 6 presents the efficiency of each independent observation under SER and ACP. One-tailed permutation tests show that the efficiency under SER is greater than the efficiency under ACP, with p = 0.05 under SYN_r, p = 0.05 under ASYN_r, p < 0.01 under SYN_f, and p < 0.01under ASYN_f.

Although both games are dominance-solvable and the amount of equilibrium play is not statistically different under complete information, their performance does differ dramatically in distributed settings with limited information and asynchrony: the serial mechanism performs robustly better than the ACP mechanism both in terms of Nash and Stackelberg equilibrium play and system efficiency.

One of the characteristics of distributed systems is the asynchrony of actions. In the following result I examine the effects of asynchrony on the proportion of equilibrium play and efficiency.

RESULT 5 (Effects of Asynchrony) : The proportion of Nash equilibrium play under SYN is significantly higher than the proportion of Stackelberg equilibrium play under ASYN. Efficiency under SYN and ASYN is not significantly different.

SUPPORT. Table 5 presents the proportion of Nash and Stackelberg equilibrium play for each independent observation. One-tailed t-test (H_0 : SYN = ASYN; H_1 : SYN > ASYN) yields z = 2.18 (p < 0.05). Table 6 presents the efficiency of each independent observation under SYN and ASYN. One-tailed t-test yields z = 1.26 (p > 0.10).

Intuitively, under the asynchronous treatments the Stackelberg leaders moved five times slower than the followers. Therefore they did not have the same opportunity to learn the equilibrium strategies. It is interesting to note that even though we observe significantly more equilibrium play in the synchronous case, the presence of asynchrony does not reduce the system efficiency significantly.

Results in this section lend support for the following result:

RESULT 6 (O-solvable vs. D-solvable Mechanisms) : The SER mechanism, which is O-solvable, performs significantly and robustly better than the ACP mechanism, which is D-solvable but not O-solvable, in terms of efficiency and the proportion of equilibrium play.

Results in Sections 4 and 5 provide empirical support for Friedman and Shenker's (1998) argument that traditional solution concepts such as Nash equilibrium or dominance-solvability are not adequate for predicting what can happen in distributed systems. Analysis of experimental data shows that O-solvable games exhibited rapid and robust convergence to the unique Nash equilibrium regardless of the degree of asynchrony, while D-solvable games did not converge as well. In Chen and Khoroshilov (2000) we examine the learning dynamics induced by the two mechanisms by comparing the explanatory power of three learning models. In Section 6 I examine whether the experimental results in the last two sections are robust in more general environments.

6 Robustness of Experimental Results in More General Environments

In this section I assess the extent to which the experimental results in Sections 4 and 5 depend on the linearity of the utility function and the quadratic cost function employed. I consider nine different environments. For simplicity I use polynomial utility and cost functions. The utility function is $\pi_i(x_i, q) = \alpha_i q_i^b - x_i$, where $\alpha_1 = 16.1$, $\alpha_2 = 20.1$ are agents' marginal utility for the output, b = 0.5, 1, and 2, and x_i is her cost share. The cost function is chosen to be $C(q) = q^c$, where c = 0.5, 1 and 2. Varying parameters b and c will give us nine combinations of concave, linear and convex utility and cost functions. Note that b = 1 and c = 2 is the original experimental design.

[Table 7 about here.]

Table 7 presents the Nash equilibrium quantities and payoffs for the two types of players under each of the nine environments. Note that all 14 boundary Nash equilibrium, (12, 12), and one

interior Nash equilibrium under SER, (2, 2), are dominant strategy Nash equilibrium, whereas the other three interior Nash equilibria are dominance solvable.

For the complete information, Round Robin treatment, I expect Result 1 to hold in each of the nine environments, i.e., the proportion of Nash equilibrium play will be indistinguishable between SER and ACP, since both games have either a dominance-solvable or a dominant strategy equilibrium in each of the nine environments, and the presentation in the form of bimatrix games is fairly transparent. Under complete information I expect that adaptive learning leads to convergence to the unique Nash equilibrium.

Note that in Table 7 the Nash equilibrium payoffs for the players are the same under the two mechanisms in eight out of nine environments, where the Nash equilibria are symmetric. This is because SER and ACP allocate the same cost share to each player when they demand the same quantity. Indeed, the only environment where the payoffs differ is the experimental environment (b = 1 and c = 2). Therefore, I expect that the efficiency will be indistinguishable between SER and ACP in each of these eight environments under complete information and Round Robin treatment. That is, Result 2 might not hold in these eight environments. This is not surprising, since in general there is no systematic efficiency comparison between the two mechanisms, as I discussed in Section 2. Therefore, any efficiency comparison between the two mechanisms will necessarily depend on the environment.

To assess how robust the experimental results are in distributed systems in different environments, I conduct Monte Carlo simulations for each of the nine environments. Since Chen and Khoroshilov (2000) study the learning dynamics induced by the SER and ACP mechanisms, I use the learning algorithm that performs the best on these data set and the calibrated parameters in Chen and Khoroshilov (2000) to conduct simulations.

Chen and Khoroshilov (2000) study how human subjects learn under extremely limited information. They use experimental data on cost sharing games reported in this paper, and Van Huyck, Battalio and Rankin's (1996) data on coordination games to compare three payoff-based learning models: the payoff-assessment learning model (Sarin and Vahid 1999), a modified experienceweighted attraction learning model (Camerer and Ho 1999) and a simple reinforcement learning model. They show that the payoff-assessment learning model tracks the data the best in both the cost sharing games as well as the coordination games. Therefore, I use the payoff-assessment learning model and the parameters calibrated on the cost sharing games to conduct simulation in other environments. Admittedly, even though the payoff-assessment learning model performs the best in capturing how human subjects learn under limited information in one environment (b = 1 and c = 2), it is possible that it might not be the best learning model when the environment changes. However, this is the best approximation we have. At least, the simulation results can show us the relative performance of the two mechanisms in other environments *if* agents are myopic maximizers described by the payoff-assessment algorithm.

The **payoff-assessment** learning model assumes that a player is a myopic subjective maximizer. She chooses among alternate strategies only on the basis of the payoff she assesses she would obtain from them. These assessments do not explicitly take into account her subjective judgements regarding the likelihood of alternate states of the world. At each stage, the player chooses the strategy that she myopically assesses to give her the highest payoff and updates her assessment adaptively. Let $u_j(t)$ denote the subjective assessment of strategy s_j at time t. The initial assessments are denoted by $u_j(0)$. Payoff assessments are updated by taking a weighted average of her previous assessments and the objective payoff she actually obtains at time t. If strategy k is chosen at time t, then

$$u_j(t+1) = (1 - rf_{jk}(h,t))u_j(t) + rf_{jk}(h,t)\pi_k(t), \forall j.$$
(1)

Suppose that at time t the decision-maker experiences zero-mean, symmetrically distributed shocks, $Z_j(t)$ to her assessment of the payoff she would receive from choosing strategy s_j , for all s_j . Denote the vector of shocks by $Z = (Z_1, \dots, Z_{12})$, and their realizations at time t by $z(t) = (z_1(t), \dots, z_{12}(t))$. The decision maker makes choices on the basis of her shock-distorted subjective assessments, denoted by $\tilde{u}(t) = u(t) + Z(t)$. At time t she chooses strategy s_j if

$$\tilde{u}_j(t) > \tilde{u}_l(t), \forall s_l \neq s_j.$$
(2)

Note that mood shocks only affect her choices and not the manner in which assessments are updated. Sarin and Vahid (1999) prove that such a player converges to stochastically choose the strategy that first order stochastically dominates another among the strategies she converges to play with positive probability.

For parameter estimation, Chen and Khoroshilov (2000) conducte Monte Carlo simulations designed to replicate the characteristics of each of the experimental settings. They then compare the simulated paths with the actual paths of a subset of the experimental data to estimate the parameters which minimize the mean-squared deviation scores. I use these estimated parameters to conduct Monte Carlo simulations for each of the nine environments.

In each simulation, 10,000 pairs of players were created¹⁷. In each simulation the following steps were taken:

¹⁷This yields a statistical accuracy of 1%.

- Initial values: Since Kolmogorov-Smirnov tests of the initial choice distribution by experimental subjects cannot reject the null hypothesis of uniform distribution. I set u_j(0) = 200 for all players when b = 0.5 and 1, since in the experimental data the average first-round payoffs was around 200 which also result in a probability predictions around the centroid, (1/12,...,1/12), for the first round. With concave (b = 0.5) and linear (b = 1) utility functions, the magnitude of payoffs are similar to the experimental setting. With convex utility function (b = 2), the payoffs are about two orders of magnitude larger than the payoff matrices in the experiment, therefore I set u_j(0) = 3000 for all players when b = 2.
- 2. Simulated players were matched into fixed pairs, or randomly rematched pairs for each period, depending on the treatment.
- 3. Shocks are drawn from a uniform distribution, [-a, a], where a is estimated¹⁸ in Chen and Khoroshilov (2000).
- 4. The simulated players' strategies were determined via Eq. (2).
- 5. Payoffs were determined using the SER or ACP payoff rule for each (b, c) parameter combination.
- 6. Assessments were updated according to Eq. (1), using discount factor, r, estimated¹⁹ in Chen and Khoroshilov (2000). Updating occurs every period under SYN for both players, every period for player 1 in ASYN and every five periods for player 2 in ASYN.

Figure 1 shows the simulated time series paths for player 1 in an environment with concave utility function (b = 0.5) under SERSYN_f (left column) and ACPSYN_f (right column) respectively. Simulated paths for player 2 exhibit similar patterns, therefore are not displayed. The first row presents the simulated paths under concave utility (b = 0.5) and concave cost function (c = 0.5). The second row presents the same information under a linear cost function (c = 1). The last row presents the same information under a convex cost function (c = 2). Each graph presents the mean

¹⁸The best fit parameters are a = 3 for SER SYN, a = 2 for SER ASYN, a = 50 for ACP SYN, and a = 45 for ACP ASYN. The larger a in ACP reflects the relatively volatile paths of the ACP data.

¹⁹The best fit parameters are r = 0.2 for SER SYN, r = 0.0 for SER ASYN, r = 0.6 for ACP SYN, and r = 0.2 for ACP ASYN.

(the black dots), standard deviation (the grey error bars) and stage game equilibria (the dashed lines) for each mechanism. Larger error bars indicate more variance in the choice of strategies and thus worse convergence to the mean. Figures 2 and 3 present the simulated time series paths for player 1 under SERSYN_f (left column) and ACPSYN_f (right column) in environments with linear (b = 1) and convex (b = 2) utility functions respectively. Simulation results for the random matching treatments display similar patterns. Therefore they are not displayed.

Based on the Monte Carlo simulation I have the following results:

- 1. With concave (b = 0.5, e.g. figure 1) and linear (b = 1, e.g. figure 2) utility functions, regardless of the forms of the cost function, simulated players converge much more quickly to the stage game equilibrium under SER than under ACP. As a result the proportion of equilibrium play is significantly larger under SER than under ACP.
- 2. With convex (b = 2, e.g. figure 3) utility function, regardless of the forms of the cost function, simulated players under both mechanisms converge quickly to the stage game equilibrium. Convergence under SER is slightly more quickly than that under ACP. As a result the proportion of equilibrium play is weakly larger under SER than under ACP.
- 3. As a result of the different speed of convergence to equilibrium, with concave and linear utility functions, efficiency under SER is significantly high than that under ACP. With convex utility functions, efficiency under SER is weakly higher than that under ACP.

Simulation results for nine different environments suggest that the experimental results on the proportion of equilibrium play are robust to variations in the environment, while experimental results on efficiency might depend on variations in the environment. In other words, even though efficiency comparison might be sensitive to the environment, the SER mechanism is more predictable than ACP because it induces robustly quicker convergence to the stage game equilibrium.

7 Conclusion

Cost sharing mechanisms have many practical applications in the real world. An increasingly important area is distributed systems like the Internet, where agents have very limited information about the payoff structure as well as the characteristics of other agents and where there is no synchronization of actions. Most current Internet routers use the average cost pricing mechanism, while this study suggests that the serial mechanism might be a better choice.

This paper reports experimental results on the serial and the average cost pricing mechanisms under five different treatments. The first is a complete information treatment designed to test the basic properties of the mechanisms. The other four treatments simulate distributed systems by giving the subjects very limited information about the game and by imposing two levels of asynchrony. The latter present a more challenging and realistic setting for the cost sharing mechanisms.

The experimental data show that under the complete information treatment both mechanisms converge well to the Nash equilibrium prediction and their performances are statistically indistinguishable. Under the limited information treatments, however, the serial mechanism performs significantly and robustly better than the average cost pricing mechanism, in terms of efficiency and convergence to equilibrium predictions regardless of the level of asynchrony. To test the robustness of the results, I conduct Monte Carlo simulation using calibrated learning algorithms in nine different environments. Simulation results indicate that the experimental results on the proportion of equilibrium play are robust to variations in the environment, while experimental results on efficiency might depend on variations in the environment.

Since both the serial and average cost pricing games are dominance-solvable in our design, these results indicate that traditional solution concepts such as Nash equilibrium or dominance-solvability might not be so useful in distributed systems. Experimental data provide empirical support for Friedman and Shenker's (1998) serially unoverwhelmed set.

References

Aadland, David and Van Kolpin. "Shared Irrigation Costs: An Empirical and Axiomatic Analysis." *Mathematical Social Sciences* 849 (1998): 203-218.

Chen, Yan and Yuri Khoroshilov. "Learning under Limited Information." Manuscript. University of Michigan, 2000.

Chen, Yan and Fang-Fang Tang. "Learning and Incentive Compatible Mechanisms for Public Goods Provision: An Experimental Study." *Journal of Political Economy* vol. 106, no. 3 (June 1998): 633-662.

Dorsey, Robert and Laura Razzolini. "An Experimental Evaluation of the Serial Cost Sharing Rule." Manuscript. University of Mississippi, 1999.

Friedman, Eric and Hervé Moulin. "Three Methods to Share Joints Costs or Surplus." *Journal of Economic Theory* vol. 87, no. 2 (1999): p.275-312.

Friedman, Eric and Scott Shenker. "Learning and Implementation on the Internet." Manuscript. New Brunswick: Rutgers University, Department of Economics, 1998.

Greenwald, Amy, Eric Friedman and Scott Shenker. "Learning in Network Contexts: Experimental Results from Simulations." *Games and Economic Behavior* forthcoming.

Kolpin, Van and David Aadland. "Environmental Determinants of Cost Sharing: an Application to Irrigation." Manuscript. University of Oregon, 2001.

Milgrom, Paul and John Roberts. "Rationalizability, Learning and Equilibrium in Games with Strategic Complementarities." *Econometrica* 58, no. 6 (1990): 1255-1277.

Moulin, Hervé and Scott Shenker. "Serial Cost Sharing." *Econometrica* 60 (September 1992): 1009-1037.

Moulin, Hervé and Scott Shenker. "Average Cost Pricing versus Serial Cost Sharing: An Axiomatic Comparison." *Journal of Economic Theory* 64 (1994): 178-201.

Shenker, Scott. "Making Greed Work in Networks: A Game-Theoretic Analysis of Gateway Service Disciplines." Manuscript. Palo Alto: Xerox Research Center, 1990.

Stoica, Ion, Scott Shenker and Hui Zhang. "Core-Stateless Fair Queueing: Achieving Approximately Fair Bandwidth Allocations in High Speed Networks." *Proceedings of ACM SIGCOMM* 1998: p.118-130.

Tauman, Y. "The Aumann-Shapley Prices: A Survey." in Alvin Roth (ed) *The Shapley Value*, New York: Cambridge University Press.

Watts, Alison. "Uniqueness of Equilibrium in Cost Sharing Games." Manuscript. Nashville: Vanderbilt University, Department of Economics, 1997.

APPENDIX. EXPERIMENT INSTRUCTIONS

Instruction for Mechanism S corresponds to the serial mechanism under Round Robin. Instruction for Mechanism A corresponds to the average cost pricing mechanism under Round Robin. Instruction for Mechanism XY is for SYN for both mechanisms, as well as for player 1 in ASYN. Instruction for Mechanism XYZ is used for player 2 in ASYN for both mechanisms.

Experiment Instructions – Mechanism S ID=

Procedure

• Each participant has to make a decision in each of _____ rounds.

• There are two different types: _ participants are Blue players and _ are Red players.

• The small envelope has your type and ID number. Your type remains the same for the entire experiment.

• A Blue player always meets a Red player and a Red player always meets a Blue player.

• In each round, a Blue is matched with a Red. You will be matched with each participant of the other type only once.

• In each round, a Blue and a Red simultaneously choose a number out of the following numbers: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, which denotes **Your Demand** of output.

• Your Payoff = (Your Valuation - Your Share of the Cost) $\times 10$.

• One unit of output is worth 16.1 points (Unit Value) to a Blue player, and 20.1 points (Unit Value) to a Red player. Therefore,

Your Valuation = (Your Unit Value) × (**Your Demand**)

• Cost of producing x units is x^2 . The smaller demander will pay half of the cost of producing twice the amount of the smaller demand, i.e., you pay the per capita cost of production as if everyone demanded the same amount as you did. Therefore, if your demand is smaller than your opponent's demand, your share of the cost is the following

Smaller Demander's Cost =
$$\frac{1}{2} \times (2 \times \text{Smaller Demand})^2$$

= $2 \times (\text{Smaller Demand})^2$

If you demand a larger amount than your opponent, you are the larger demander. You will pay the rest of the cost of production. Therefore,

Larger Demander's Cost =
$$\underbrace{(\text{Smaller} + \text{Larger Demand})^2}_{\text{Total Cost of Production}} - \underbrace{2 \times (\text{Smaller Demand})^2}_{\text{Smaller Demander's Cost}}$$

• For example, if a Blue demands 2 unit and a Red demands 1 units, then the total demand is 3 units. The calculation of payoffs for the two types are tabulated as follows:

	Blue	Red
Your Demand	2 (Larger)	1 (Smaller)
Your Valuation	$16.1 \times 2 = 32.2$	$20.1 \times 1 = 20.1$
Your Share of the Cost	$(1+2)^2 - 2 \times 1^2 = 7$	$2 \times 1^2 = 2$
Your Payoff	$(32.2 - 7) \times 10 = 252$	$(20.1 - 2) \times 10 = 181$

• The above information is summarized by the Payoff Tables in your folder.

Payoff Table

• The Payoff Table summarizes both your payoff and your opponent's payoff. A Blue player chooses which row to play. A Red player chooses which column to play. Your payoff is determined by both your choice and your opponent's choice. The first number in each cell (in blue) denotes the payoff to a Blue player. The second number in each cell (in red) denotes the payoff to a Red player.

• For example, if a Blue demands 2 unit, and a Red demands 1 unit, you can find the payoff to each participant on the second row and the first column of the payoff matrix. The cell contains two numbers: the first number is 252, which is Blue's payoff; the second number is 181, which is Red's payoff.

Information

- At the end of each round, each participant is informed of the following results of the round:
- your own demand
- your opponent's demand
- your own payoff
- the distribution of demands of the other type in the last round.
- You will not know who your opponent was.

Total Payoff

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is \$1 for ____ points.

Record Sheet: you are required to record your demand, your opponent's demand and your payoff each round.

Review Questions (Write down your answers on top of your Record Sheet.)

1. You are a ____ (Blue or Red) player.

2. If you demand 2 units and your opponent demands 10 units, your payoff is ____; your opponent's payoff is ____. Find it from your payoff table.



-																									
221	296-	802-	800-	01	800-	585	050-	1000	607-	225	905	907	100-	205	951-	101	200-	27	901-	222	25	101	292	51	
257	693-	602-	013-	01	603-	GØ5	609-	925	633-	124	033-	2017	eec.	202	035-	727	651-	627	e-	225	131	191	ISC	ħ	
829	060-	055-	060-	01	060-	685	085-	220	050-	124	005-	2017	OEZ-	505	011-	9/29	00-	627	001	en:	02	101	450	01	
87.6	121-	696-	171-	02	171-	S\$S	171-	328	131-	124	161-	2017	10-	202	11-	9/29	61	627	031	222	610	101	034	•	
100		811-	8	7 5		551	8	225		224	91	207	19	505	96	929	101	627	952	222	190	101	959	P.	
912	291	61-	291	001	291	SZZ	291	955	291	124	195	2017	291	505	291	929	20Z	627	70E	225	28C	191	705	Ł	
801	246	15	345	011	246	62Z	345	225	246	257	345	2017	246	505	325	9/29	962	627	300	222	209	101	961	Þ	100040
2	500	151	200	092	\$00	SPC	205	219	\$00	254	200	967	\$00	200	205	999	SIC	czr	245	222	505	101	239	Þ	x,orig
211	920	182	920	07.6	925	507	920	227	920	215	920	925	920	515	920	121	920	627	920	225	120	191	919	ŀ	
ъĸ	200	123	202	00%	200	545	202	01.C	200	285	202	9/5	200	595	202	404	200	CCY	202	275	cic.	101	242	ŀ.	
ZES	242	109	242	092	242	629	242	892	242	229	292	29-2	242	585	242	925	292	CC7	242	200	242	591	252	ř	
291	191	162	191	028	191	62R	191	515	191	282	191	36.1	191	500	191	925	191	C397	191	225	191	191	191	ŀ	
21		11		01		6		2		2		2		\$		2		٤.		2		÷			
											-	9043	s pag												
											şu	sion.	128W												

Experiment Instructions – Mechanism A ID=

Procedure

• Each participant has to make a decision in each of _____ rounds.

• There are two different types: _ participants are Blue players and _ are Red players.

• The small envelope has your type and ID number. Your type remains the same for the entire experiment.

• A Blue player always meets a Red player and a Red Player always meets a Blue player.

• In each round, a Blue is matched with a Red. You will be matched with each participant of the other type only once.

• In each round, a Blue and a Red simultaneously choose a number out of the following numbers: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, which denotes **Your Demand** of output.

• Your Payoff = (Your Valuation - Your Share of the Cost) $\times 10$.

• One unit of output is worth 16.1 points (Unit Value) to a Blue player, and 20.1 points (Unit Value) to a Red player. Therefore,

Your Valuation = (Your Unit Value) × (**Your Demand**)

• Cost of producing x units is x^2 . Your share of the cost is proportional to your demand. Therefore,

Your Share of the Cost =
$$\frac{\text{Your Demand}}{\text{Total Demand}} \times (\text{Total Demand})^2$$

= (Your Demand) × (Total Demand),

where Total Demand = Your Demand + Your Opponent's Demand.

• For example, if a type A demands 2 unit and a type B demands 1 units, then the total demand is 3 units and the total cost of producing 3 units is 9. The calculation of payoffs for the two types are tabulated as follows:

	Blue	Red
Your Demand	2	1
Your Valuation	$16.1 \times 2 = 32.2$	$20.1 \times 1 = 20.1$
Your Share of the Cost	$\frac{2}{3} \times 9 = 6$	$\frac{1}{3} \times 9 = 3$
Your Payoff	$(32.2 - 6) \times 10 = 262$	$(20.1 - 3) \times 10 = 171$

• The above information is summarized by the Payoff Table in your folder.

Payoff Table

• The Payoff Table summarizes both your payoff and your opponent's payoff. A Blue player chooses which row to play. A Red player chooses which column to play. Your payoff is determined by both your choice and your opponent's choice. The first number in each cell (in blue) denotes the payoff to a Blue player. The second number in each cell (in red) denotes the payoff to a Red player.

• For example, if a Blue demands 2 unit, and a Red demands 1 unit, you can find the payoff to each participant on the second row and the first column of the payoff matrix. The cell contains two numbers: the first number is 262, which is Blue's payoff; the second number is 171, which is Red's payoff.

Information

- At the end of each round, each participant is informed of the following results of the round:
- your own demand
- your opponent's demand
- your own payoff
- the distribution of demands of the other type in the last round.
- You will not know who your opponent was.

Total Payoff

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is \$1 for ____ points.

Record Sheet

• You are required to record your demand, your opponent's demand and your payoff each round.

Review Questions (Write down your answers on top of your Record Sheet.)

1. You are a ____ (Blue or Red) player.

2. If you demand 2 units and your opponent demands 10 units, your payoff is ____; your opponent's payoff is ____. Find it from your payoff table.



27	896-	810-	858-	061-	807-	18-	005-	2	199-	22	895.	221	822-	555	801-	ня.	21	655	201	21	292	12	220	21	
90	ect.	sce-	013-	06-	603-	5	621-	22	e1C-	29%	602-	201	60-	soe	11	902	121	ces	102	295	190	10	159	ŀ	
125-	062-	68-	-420	01	060-	33	052-	201	061-	212	05-	242	01	552	011	792	012	512	015	231	019	81	015	01	
101-	199-		190-	011	192-	oes	121-	arc.	10-	282	6	205	66	206	681	992	61Z	542	696	cu:	657	501	695	ŀ	
51	210-	121	202-	510	251-	64Z	57.	azc	8	2SE	92	201	191	SSE	348	224	920	£22	909	202	229	111	835	Þ	
T 1	£05-	152	221-	210	23-	69C	L	907	ш	124	291	929	212	509-	782	920	73E	coc	724	222	2011	121	738	k	
52	111-	INC	99°	019	2	420	22	227	921	255	221	2017	246	462	305	101	225	222	305	242	207	101	345	þ	180/04
212	511-	159	s	015	55	615	901	225	551	199	305	285	59Z	505	90C	***	590	C9C	909	C3C	559	191	909	Þ	1,010
107	۲	195	99	019	90	809	121	29-2	¥91	259	102	202	HC	555	992	929	125	cec	990	CW2	101	151	***	ŀ	
19	22	129	63	01 Z	52	572	521	97.L	CSI.	202	281	222	212	500	542	925	£12	622	202	cα	222	191	230	k	
<u>81</u>	٢r	192	29	018	28	518	201	202	152	ш	291	35.1	291	555	281	125	202	C59/	222	205	242	123	292	Ł	
598	10	168	19	810	19	808	19	002	12	299	12	287	16	902	101	102	***	C29	121	2M2	101	181	191	ŀ	
51		14		01		5	_		_	.	_	-	_	-	_	Ļ.	_	-	_	7	_	÷	_	T.	

Mechani

Experiment Instructions – Mechanism XY ID=

Procedure

- You are part of a game, in which you have to make a decision in each of 150 rounds.
- In each round, you will choose a number out of the following numbers:

 $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$

Information

- At the end of **each round**, you are informed of your result of the round:
- your own choice
- your own payoff

Total Payoffs

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is \$1 for ____ points.

Record Sheet: you are required to record your choice and your payoff each round.

Experiment Instructions – Mechanism XYZ ID=

Procedure

- You are part of a game, in which you have to make a decision in each of 30 rounds.
- In each round, you will choose a number out of the following numbers:

 $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$

Information

- At the end of each round, you are informed of your result of the round:
- your own choice
- your own payoff

Total Payoffs

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is \$1 for ____ points.

Record Sheet: you are required to record your choice and your payoff each round.

		Equilibrium Quantities							
ID	Parameters		SER	ACP					
	α_i	Nash	Stackelberg	Nash	Stackelberg				
			(Leader: 1 or 2)		(Leader: 2)				
1 (Blue)	16.1	4	4	4	2				
2 (Red)	20.1	6	6	8	12				

Table 1: Parameters and Equilibrium Quantities

		# of Subjects		Game
Treatments	Session # (Subject Pool)	Per Session	Mechanisms	Length
	1, 2, 3 (CIT)	16, 16, 16	SER	
Round	4, 5, 6 (PCC)	16, 16, 16	SER	
Robin	7, 8, 9 (CIT)	16, 16, 16	ACP	8
	10, 11, 12 (PCC)	16, 16, 16	ACP	
Synchronous	13 (CIT), 14 (PCC), 15 (UM)	16, 12, 12	SER	
(fixed pair)	16 (CIT), 17 (PCC), 18 (UM)	14, 12, 12	ACP	
Synchronous	19, 20, 21(UM)	12, 12, 12	SER	
(random match)	22, 23, 24 (UM)	12, 12, 12	ACP	
Asynchronous	25 (CIT), 26 (PCC), 27 (UM)	10, 12, 12	SER	150
(fixed pair)	28 (CIT), 29 (PCC), 30 (UM)	12, 12, 12	ACP	
Asynchronous	31, 32, 33 (UM)	12, 12, 12	SER	
(random match)	34, 35, 36 (UM)	12, 12, 12	ACP	

Table 2: Features of Experimental Sessions

Session	Mechanism				Round	#				All
#	(Subj. Pool)	1	2	3	4	5	6	7	8	Rounds
1	SER (CIT)	.625	.688	.938	1.000	.938	1.000	1.000	1.000	.899
2	SER (CIT)	.688	.750	.938	.938	.938	.938	1.000	1.000	.899
3	SER (CIT)	.750	.938	.825	.938	1.000	1.000	1.000	1.000	.931
4	SER (PCC)	.188	.313	.438	.625	.750	.625	.825	.813	.572
5	SER (PCC)	.313	.375	.438	.313	.375	.625	.625	.688	.469
6	SER (PCC)	.125	.313	.313	.438	.500	.750	.688	.813	.493
7	ACP (CIT)	.438	.438	.688	.688	.813	.813	.825	.825	.691
8	ACP (CIT)	.375	.500	.688	.625	.750	.813	.813	.750	.664
9	ACP (CIT)	.688	1.000	1.000	.938	.938	1.000	.825	.813	.900
10	ACP (PCC)	.250	.375	.688	.625	.813	.813	.813	.825	.650
11	ACP (PCC)	.313	.438	.375	.563	.625	.813	.688	.825	.580
12	ACP (PCC)	.500	.563	.563	.750	.813	.813	.938	.813	.719
Perm. tests	p-value	.461	.484	.549	.481	.667	.656	.323	.083	.456

Table 3: Proportion of Subjects Choosing Nash Equilibrium Strategies under the Round Robin Treatment and Results of Permutation Tests (H_0 : SER = ACP; H_1 : SER > ACP.)

Session #	Mechanism	Subj. Pool	Efficiency
1.	SER	CIT	.8669
2.	SER	CIT	.8655
3.	SER	CIT	.8696
4.	SER	PCC	.8241
5.	SER	PCC	.8069
6.	SER	PCC	.8206
7.	ACP	CIT	.8066
8.	ACP	CIT	.7809
9.	ACP	CIT	.8326
10.	ACP	PCC	.7788
11.	ACP	PCC	.8070
12.	ACP	PCC	.8310

Table 4: Efficiency of Each Session Under the Round Robin Treatment

	S	ER	A	СР
# of Sessions	SYN _r	$ASYN_r$	SYN_r	$ASYN_r$
1	0.529	0.544	0.166	0.051
2	0.669	0.447	0.149	0.045
3	0.468	0.596	0.174	0.066
# of Pairs	SYN_f	$ASYN_f$	SYN_f	$ASYN_f$
1	0.873	0.683	0.197	0.077
2	0.343	0.383	0.207	0.003
3	0.893	0.480	0.383	0.393
4	0.710	0.687	0.400	0.040
5	0.937	0.737	0.743	0.053
6	0.760	0.553	0.610	0.123
7	0.883	0.363	0.180	0.030
8	0.657	0.477	0.117	0.007
9	0.937	0.367	0.200	0.120
10	0.773	0.630	0.110	0.117
11	0.133	0.327	0.067	0.043
12	0.683	0.677	0.167	0.050
13	0.380	0.637	0.107	0.077
14	0.283	0.317	0.160	0.020
15	0.690	0.557	0.143	0.087
16	0.767	0.647	0.047	0.363
17	0.503	0.703	0.147	0.040
18	0.590		0.170	0.027
19	0.847		0.060	
20	0.737			

Table 5: Proportion of Nash/Stackelberg Equilibrium Play in Distributed Systems

	S	ER	A	СР
# of Sessions	SYN_r	$ASYN_r$	SYN_r	$ASYN_r$
1	0.800	0.797	0.607	0.690
2	0.821	0.779	0.612	0.690
3	0.793	0.799	0.658	0.625
# of Pairs	SYN_f	$ASYN_{f}$	SYN_f	$ASYN_f$
1	0.853	0.829	0.709	0.688
2	0.524	0.797	0.670	0.761
3	0.851	0.789	0.844	0.772
4	0.788	0.841	0.778	0.653
5	0.858	0.817	0.787	0.708
6	0.859	0.776	0.765	0.729
7	0.845	0.710	0.640	0.682
8	0.812	0.769	0.600	0.729
9	0.847	0.733	0.587	0.700
10	0.831	0.789	0.562	0.572
11	0.779	0.767	0.520	0.709
12	0.821	0.785	0.600	0.760
13	0.742	0.790	0.567	0.697
14	0.683	0.768	0.648	0.693
15	0.827	0.795	0.474	0.777
16	0.832	0.848	0.489	0.552
17	0.784	0.797	0.515	0.689
18	0.803		0.711	0.712
19	0.817		0.301	
20	0.832			

Table 6: Efficiency in Distributed Systems

			SER		ACP					
Parame	ters	c = 0.5	c = 1	c = 2	c = 0.5	c = 1	c = 2			
	q	(12, 12)	(12, 12)	(2, 2)	(12, 12)	(12, 12)	(1, 3)			
b = .5	π	(533,672)	(438,576)	(148,204)	(533,672)	(438,576)	(148,204)			
	q	(12, 12)	(12, 12)	(4, 6)	(12, 12)	(12, 12)	(4, 8)			
b = 1	π	(1908,2388)	(1812,2292)	(324,526)	(1908,2388)	(1812,2292)	(164,648)			
	q	(12, 12)	(12, 12)	(12, 12)	(12, 12)	(12, 12)	(12, 12)			
b=2	π	(23160,28920)	(23064,28824)	(20304,26064)	(23160,28920)	(23064,28824)	(20304,26064)			

Table 7: Nash equilibrium quantities (q) and payoffs (π) for utility function $\alpha_i q_i^b - x_i$ and cost function $C(q) = q^c$.



Figure 1. Simulation results with concave utility function concave, linear, and convex cost functions.



Figure 2. Simulation results with linear utility functions, concave, linear, and convex cost functions.

