Congestion Allocation for Distributed Networks: An Experimental Study *

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Abstract

This paper reports an experimental study of two prominent congestion and cost allocation mechanisms for distributed networks. We study the fair queueing (or serial) and the FIFO (or average cost pricing) mechanisms under two different treatments: a complete information treatment and a limited information treatment designed to simulate distributed networks. Experimental results show that the fair queueing mechanism performs significantly better than FIFO in all treatments in terms of efficiency, predictability measured as frequency of equilibrium play, and the speed of convergence to equilibrium. Monte Carlo simulations of a calibrated learning model show that the results are robust to changes in the environment with concave or linear utility functions.

Keywords: serial mechanism, congestion allocation, experiment
JEL Classification: C91, D83

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1 Introduction

There have been much interests in the problem of congestion allocation in computer networks, as the Internet becomes increasingly important in global telecommunications and e-commerce. In distributed networks such as the Internet, multiple agents share the same network link. Each agent controls the rate at which she is transmitting data. If the sum of the transmission rates is greater than the total link capacity, then the link becomes congested and the agents' packets experience delays. Most current Internet routers use a FIFO packet scheduling algorithm, where all packets are serviced on a first-come-first-serve basis. Each user's average queue is proportional to their throughput (Shenker 1990). One agent's usage can affect the quality of service of other agents. Aggressive users can get more than an equal share of these shared facilities. For example, agents who modify their Transmission Control Protocol implementation to be less responsive when congestion is detected can obtain much larger shares of the bandwidth (Demers *et al.*, 1990). In contrast, the Fair Queueing packet scheduling algorithm, originally proposed by Shenker (1990), leads to congestion allocations such that an agent's average queue is independent of transmission rates higher than her own. For example, if each user contribute an independent Poisson input stream of packets with various rates, the Fair Queueing algorithm allocates congestion by a preemptive priority queueing algorithm, where users are ordered by increasing transmission rates. All of the smallest user's packets are in the highest priority class, and all of the other users get the same rate (as the smallest user) of packets in the highest priority class. Similarly, the rest of the second smallest user's packets are in the second highest priority class, and all of the other users gets the same rate of packets in the second highest priority class; and so on. The Fair Queueing algorithm has been proposed as an alternative to the FIFO algorithm, based on theoretical and simulation results (Stoica, Shenker and Zhang, 1998). The new generation of Cisco 7200, 3600 and 2600 routers have both the FIFO and Fair Queueing options. In this paper, we evaluate the performance of these two algorithms using laboratory experiments.

Congestion allocation in distributed networks is closely related to the more general class of cost sharing problems. A cost-sharing mechanism distributes the service and allocates the corresponding costs to each agent. The FIFO packet scheduling algorithm corresponds to the average cost pricing mechanism (Shenker, 1990), where an agent's cost share is proportional to her own demand, while the Fair Queueing algorithm corresponds to the serial cost sharing mechanism.

The theoretical literature on cost sharing has largely focused on the axiomatic characterization of these mechanisms (e.g., Moulin and Shenker,1994; Friedman and Moulin, 1999) and their static properties in a complete information setting with synchronous actions. However, as Friedman and Shenker (1998) pointed out, in a *distributed system*[1] such as the Internet where agents have very limited *a priori* information about other agents and the payoff structure, traditional solution concepts might not be able to predict the outcome of learning. It is important to empirically study the actual learning dynamics among real players in settings similar to distributed networks and examine whether learning will lead to the equilibrium predicted by

[1]Following Friedman and Shenker (1998), a system is called a *distributed system* "because the users are geographically dispersed and are accessing the resource through the network." The Internet is a prominent example.

theory. This paper does this by investigating the learning dynamics induced by each mechanism under both complete and limited information settings in a laboratory environment.

We are aware of five experimental studies of cost sharing mechanisms. Chen (2003) studies the serial and average cost pricing mechanisms under complete information as well as limited information when there are two types of agents. She found that the performance of the two mechanisms are statistically indistinguishable under complete information. Under limited information, however, the serial mechanism performs robustly better than the average cost pricing mechanism in terms of frequency of equilibrium play and system efficiency. Chen and Khoroshilov (2003) study the learning dynamics in these cost sharing games and other games under limited information. Razzolini *et al.* (1999) investigate the performance of the serial mechanism with each human subject against three computerized players, where each human player knows his own cost share and payoff structure but has no information about the opponents' payoff structures. Their information condition is in between the complete information and limited information setting in Chen (2003). While in Chen's experiment, the subjects maintain their preference parameter throughout the entire experiment, in Razzolini *et al.*'s experiment, the subjects' preference parameters change in each period. This implies that in each period the allocation mechanism must converge to a different Nash equilibrium allocation. Razzolini *et al.* (1999) implement the serial mechansism both as a sequential game and as a simultaneous normal form game. They found that the serial mechanism leads to almost efficient allocations, and even though more easy to understand and implement, the simultaneous move treatment does not lead to a better overall performance. Chen (2003) uses a payoff table to explain both mechanisms, which is feasible for the serial mechanism with only two types of players. When the number of types increase, the serial mechanism becomes more challenging to implement in the laboratory, because the dimension of payoff tables increases with each additional type. With more than two types one needs to find alternative ways to implement the mechanism. Razzolini *et al.* (1999) has four different types, but only one of them is a human player, thus the strategic interaction between different types are simplified. Each of the two studies highlights different aspects of the cost sharing mechanisms. They present the first steps in understanding how these mechanisms work.

Gailmard and Palfrey (2005) report experiments for the provision of excludable threshold public goods and compare the serial cost sharing mechanism with voluntary cost sharing with proportional rebates and with no rebates. They found that voluntary cost sharing with rebates outperforms serial on welfare grounds, which in turn outperforms voluntary cost sharing with no rebates. One possible reason for the difference between Gailmard and Palfrey's results and the two previous studies might be that Gailmard and Palfrey (2005) use an excludable threshold public goods, while Chen (2003) and Razzolini *et al.* (1999) use multiple levels of private goods. Rapoport *et al.* (2001) report an experimental study of a large-scale queueing game with the FIFO queue discipline (i.e., average cost sharing mechanism). Their results show strong support for mixed strategy equilibrium play on the aggregate level but not on the individual level.

This paper is a natural extension of Chen (2003) and Razzolini *et al.* (1999). In this paper we design an experiment to evaluate the serial and the average cost pricing mechanism in a baseline complete information environment, and a more challenging environment with limited information. In our environment, there are twelve players of four different types. Thus, the environment is more complex than the two earlier studies. The goal of this paper is to assess the performance of the two mechanisms in different settings, to study how human subjects learn in these different settings, whether and how the learning dynamics leads to convergence to stage game Nash equilibrium, and ultimately test the practical implementability of the fair queuing or serial mechanism.

The paper is organized as follows. Section 2 introduces the theoretical properties of the serial (hereafter shortened as SRL) and average cost pricing (hereafter shortened as ACP) mechanisms. Section 3 presents the experimental design. Section 4 compares the performance of the mechanisms under complete information and limited information. Section 5 discusses the robustness of the experimental results with respect to changes in the environment by calibrating a learning model and using the calibrated model to forecast performance in other environments. Section 6 concludes the paper.

2 Theoretical Properties of the Mechanisms

Let $N = \{1, \cdots, i, \cdots, n\}$ be a group of agents sharing a one-input, one-output technology. Each of the n agents announces his demand q_i of output. Each agent gets her demand q_i and pays a cost share, x_i. Note x_i is the total cost agent i pays. In the example of Internet routers, q_i is agent i's data transmission rate, while x_i is the reduction in agent i's utility due to congestion. Let $q_1 \leq q_2 \leq \cdots \leq q_n$. The cost function is denoted by C, which is strictly convex. A cost-sharing mechanism must allocate the total cost $C(\sum_i q_i)$ among the n agents.

The serial mechanism, originally introduced by Shenker (1990), was analyzed by Moulin and Shenker (1992) in the context of cost and surplus sharing with complete information. The mechanism can be characterized by four properties: unique Nash equilibrium at all profiles[2], anonymity (the name of the agents does not matter), monotonicity (an agent's cost share increases when she demands more output) and smoothness (an agent's cost share is a continuously differentiable function of the vector of demands). Among agents endowed with convex, continuous and monotonic preferences, the serial mechanism is the only cost sharing rule which is dominance-solvable and its unique Nash equilibrium is also robust to coalitional deviations when agents cannot transfer outputs.

Under the serial mechanism, agent 1 (with the lowest demand) pays $(1/n)$th of the cost of producing nq_1, $x_1^s = C(nq_1)/n$. Agent 2 pays agent 1's cost share plus $1/(n-1)$th of the incremental cost from nq_1 to $(n-1)q_2 + q_1$, i.e.,

$$x_2^s = \frac{C(nq_1)}{n} + \frac{C(q_1 + (n-1)q_2) - C(nq_1)}{n-1}.$$

And so on. Let $q^0 = 0$; $q^1 = nq_1$; $q^2 = q_1 + (n-1)q_2; \cdots$; $q^i = q_1 + \cdots + q_{i-1} + (n+1-i)q_i; \cdots,$ $q^n =$

[2] Assume agents have convex, continuous and monotonic preferences.

$\sum_i q_i$. Then the general formula for agent i's cost share is given below,

$$x_i^s(c,q) = \sum_{k=1}^{i} \frac{C(q^k) - C(q^{k-1})}{n+1-k}, \text{ for all } i = 1, \cdots, n.$$

Therefore, an agent's cost share under the serial mechanism is only affected by her own demand and those whose demands are lower than hers. In other words, an agent's cost share is independent of demands higher than her own.

Like the serial mechanism, the average cost pricing mechanism satisfies anonymity, monotonicity and smoothness. It is the only method that is robust to arbitrage, i.e., agents cannot benefit from merging or splitting their demands. In contrast to the serial mechanism, the normal form game induced by the average cost pricing mechanism is in general not dominance-solvable, nor does it have a unique equilibrium at all profiles when agents have convex, continuous and monotonic preferences.

When agent i demands q_i amount of output, the general formula for agent i's cost share under the average cost pricing mechanism is given by

$$x_i^a(c,q) = (q_i / \sum_k q_k) \cdot C(\sum_k q_k), \text{ for all } i = 1, \cdots, n.$$

Therefore, under ACP an agent's cost share is proportional to her demand. It is affected by her own demand, and the sum of all other agents' demands.

There is no systematic efficiency comparison between the two mechanisms. In general there exists no differentiable and monotonic cost sharing mechanism where all Nash equilibrium outcomes are first best Pareto optimal at all preference profiles. Moulin and Shenker (1992) provide a definition of second best efficiency[3] and show that the serial mechanism yields a second best efficient equilibrium while ACP does not.

In distributed systems, such as the Internet where users are geographically dispersed and have little information about other players and the payoff structure, for learning to converge to equilibrium, it is important that strategies sampled by players are informative. In this respect, the serial mechanism has an advantage over ACP. We first define some new concepts. In a normal form game, one action *overwhelms* another if all payoffs, over all sets of other players' actions, for the one are greater than all payoffs, over all sets of other players' actions, for the other[4]. The *serially unoverwhelmed set* is the set remaining after iterated elimination of overwhelmed actions. A game is *D-solvable* if iterated elimination of dominated strategies leads to a single eventual outcome. A game is *O-solvable* if iterated elimination of overwhelmed strategies leads to a single eventual outcome. Friedman and Shenker (1998) prove that reasonable learners[5] converge

[3]"For an arbitrary cost sharing mechanism ξ, say that $(q_1, \cdots, q_n)$ is a Nash equilibrium outcome at some utility profile. We ask if there is another vector of demands $(q_1^{'}, \cdots, q_n^{'})$ such that at the corresponding allocation dictated by the mechanism ξ, no one is worse off and someone is better off than at the equilibrium allocation corresponding to $(q_1, \cdots, q_n)$. If no such vector of demands exists, we call our equilibrium second best efficient." Moulin and Shenker (1992, p.1025)

[4]See Friedman and Shenker (1998) for a precise definition.

[5]The key components of a reasonable learner are optimization, monotonicity and responsiveness. See Friedman and Shenker (1998).

to the serially unoverwhelmed set. In comparison, Milgrom and Roberts (1990) showed that adaptive learners converge to the serially undominated set. Among the cost sharing mechanisms, the serial mechanism is O-solvable[6] while ACP is not. If a strategy overwhelms another one, sampling is much more informative than situations where one strategy dominates another, as the minimum payoff from the overwhelming strategy is at least as large as the maximum payoff of the overwhelmed strategy. In environments with limited information, informative sampling can significantly increase the speed of learning.

3 Experimental Design

The goal of the experimental design is to compare the performance of the SRL and ACP mechanisms in two different settings: a complete information setting that tests the prediction of dominance-solvability, and a more challenging network setting to compare the performance of the two mechanisms and to assess the plausibility of the new solution concepts. The economic environment and experimental procedures are discussed in the sections below.

3.1 The Economic Environment

In a simple environment to test the serial and ACP mechanism under various treatments, agents are endowed with linear preferences $\pi_i(x_i, q) = \alpha_i q_i + \omega_i - x_i$, where α_i is agent i's marginal utility for the output, ω_i is agent i's lump-sum endowment and x_i is her cost share. The cost function is chosen to be quadratic, $C(q) = q^2$. We call this environment E. In the network context with several agents sharing a network link, α_i is agent i's value for the amount of data transmitted per unit of time, and the cost to be allocated corresponds to the congestion experienced. Therefore, the cost should be interpreted as the reduction in agent i's utility due to congestion. We chose linear utility and quadratic cost functions in order to get an interior integer Nash equilibrium. In Section 5 we present simulation results for more general utility and cost functions.

Consider a four-player game with $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4$. Under the serial mechanism, the cost share for agent 1 is $x_1^s = C(4q_1)/4$. Agent 2's cost share is $x_2^s = x_1^s + (C(q_1 + 3q_2) - C(4q_1))/3$. Agent 3's cost share is $x_3^s = x_2^s + (C(q_1 + q_2 + 2q_3) - C(q_1 + 3q_2))/2$. Agent 4's cost share is $x_4^s = x_3^s + (C(q_1 + q_2 + q_3 + q_4) - C(q_1 + q_2 + 2q_3))$. When agents maximize their utility over a continuous strategy space, the unique, dominance-solvable Nash equilibrium is characterized by

$$q_1^s = \frac{\alpha_1}{8},\ \ q_2^s = \frac{\alpha_2}{6} - \frac{\alpha_1}{24},\ \ q_3^s = \frac{\alpha_3}{4} - \frac{\alpha_1}{24} - \frac{\alpha_2}{12},\ \text{and}\ q_4^s = \frac{\alpha_4}{2} - \frac{\alpha_1}{24} - \frac{\alpha_2}{12} - \frac{\alpha_3}{4}.$$

For the ACP mechanism, the cost shares of each of the four agents are $x_i^a = \frac{q_i}{\sum_{i=1}^4 q_i} C(\sum_{i=1}^4 q_i) = q_i(\sum_{i=1}^4 q_i)$. Even though the normal form game induced by ACP is in general not dominance solvable, nor does it have a unique equilibrium at all profiles, in our experimental environment it is dominance solvable

[6] This is proved in Theorem 8 in Friedman and Shenker (1998).

and has a unique equilibrium when the strategy space is continuous. The unique, dominance-solvable Nash equilibrium is characterized by

$$q_i^a = \frac{4\alpha_i}{5} - \frac{\sum_{j\neq i}\alpha_j}{5}, \forall i.$$

The mechanisms are implemented as normal form games with a discrete strategy space for each player, $\{0, 1, \cdots, 19, 20\}$. Parameters are chosen to ensure: (1) With a continuous strategy space, the SRL game is both D-solvable and O-solvable, while the ACP game is D-solvable but not O-solvable; (2) Nash equilibrium strategies are all integers; (3) most of the payoffs are positive in both normal form games; (4) lump-sum payments are allocated in a way that the sum of all players' Nash equilibrium payoffs in the SRL and ACP games are the same with a continuous strategy space. This enables us to make efficiency comparisons between the two mechanisms.[7] (5) Within each game the lump-sum payoffs are allocated such that the equilibrium payoffs are not too skewed among different types of players.

Based on the theoretical properties of the mechanisms and the design, we expect the performance of the two mechanisms to be the same under complete information. Under limited information, however, we expect SRL to have higher level of equilibrium play (and hence higher efficiency), as well as faster convergence to equilibrium. We formally state the following hypotheses.

HYPOTHESIS 1 *Under complete information, SRL and ACP will generate the same proportion of equilibrium play.*

HYPOTHESIS 2 *Under limited information, SRL will generate a higher proportion of equilibrium play than ACP.*

HYPOTHESIS 3 *Under complete information, the speed of convergence is the same under SRL and ACP.*

HYPOTHESIS 4 *Under limited information, the speed of convergence is faster under SRL than under ACP.*

HYPOTHESIS 5 *Under complete information, SRL and ACP will generate the same level of efficiency.*

HYPOTHESIS 6 *Under limited information, SRL will generate higher efficiency than ACP.*

[Table 1 about here.]

Table 1 reports the parameters, equilibrium quantities and payoffs for the two mechanisms. Note that we use Blue for player 1, Green for player 2, Red for player 3 and Yellow for player 4 in the instructions (see Appendix A). In the columns under Equilibrium Quantities, the SRL mechanism still has a unique Nash equilibrium as is the case with a continuous strategy space, $(6, 7, 8, 9)$. Under ACP, however, apart from the unique Nash equilibrium with a continuous strategy space, $(4, 10, 14, 16)$, there are eighteen additional Nash equilibria when the strategy space is discrete.

[7] Note that generically in the same economic environment in Nash equilibrium either the SRL or the ACP game yields higher aggregate payoffs, making it inappropriate to do an efficiency comparison.

[Table 2 about here.]

Table 2 lists all nineteen Nash equilibria for the ACP game. They are organized by the equilibrium quantities from the smallest demander to the largest demander. Equilibrium number 10 (in bold) is the original Nash equilibrium of the continuous game. Note all 19 equilibria have the same aggregate demand, $\sum_{i=1}^{4} q_i^a = 44$. The last column lists the aggregate equilibrium payoffs in decreasing order. Next we show that multiple equilibria as a result of discretization is a generic property of the average cost pricing mechanism, regardless of the step size for discretization. Let D be a discrete strategy space such that the equilibrium of the continuous strategy space, $\{q_i^*\}_{i\in N} \in D$. Let $s > 0$ be the step size in D. The following proposition characterizes the Nash equilibria of the ACP mechanism with a discrete strategy space D.

PROPOSITION 1 *In environment E under the ACP mechanism, if $\{q_i^*\}_{i\in N}$ is the unique Nash equilibrium of the continuous game, then $\{\bar{q}_1, \cdots, \bar{q}_n | \bar{q}_i \in \{q_i^* - s, q_i^*, q_i^* + s\}$ and $\sum_i \bar{q}_i = \sum_i q_i^*, \forall i \in N, \forall s > 0\}$ are all Nash equilibria of the discrete game.*

Proof: see Appendix B.

Even though Proposition 1 only deals with our experimental environment of linear preferences and quadratic cost functions, multiple equilibria with discretization is a generic problem with the ACP mechanism. We will discuss the multiple equilibria problem in other environments in Section 5.

3.2 Experimental Procedures

We implemented a 2×2 factorial design by varying the mechanisms and information conditions. We conducted five independent sessions for each of the four treatments. Each session had twelve subjects and last for fifty rounds. Players always kept their own type. For a baseline comparison, we conducted ten sessions of the SRL and ACP mechanisms under complete information with the random matching protocol (hereafter shortened as SRL$_c$ and ACP$_c$). Under complete information, each player was informed of the payoff matrix, the structure of the game, matching protocols, the quantities chosen and the corresponding payoffs earned by all players in all rounds. This pair of treatments were designed to compare the performance of the two mechanisms as one-shot games under complete information. The natural solution concept for these treatments is dominance-solvability. To evaluate the possibility of applying these mechanisms to distributed systems such as the Internet, we designed a pair of limited information treatments. Learning in distributed systems is characterized by the feature that players might have extremely limited information. They often do not know the payoff functions, nor do they know how their payoffs depend on the actions of others, probably due to the lack of information about the detailed nature of the resources itself. Therefore, in the limited information treatments, the only information players had was their own action and the resulting own payoffs. In the limited information treatments, players were again randomly re-matched into groups of four in each round (hereafter shortened as SRL$_l$ and ACP$_l$).

Computerized experiments were conducted at the RCGD Laboratory at the University of Michigan in July and August, 2001. We conducted twenty independent sessions. Subjects were students from the

University of Michigan. A total of 240 subjects participated in the experiment. No subject was used in more than one session.

[Table 3 about here.]

Table 3 summarizes features of the experimental design. At the beginning of each session subjects randomly drew a PC terminal number. Then each of them was seated in front of the corresponding terminal, and given the instructions. After the instructions were read aloud, subjects were required to finish the Review Questions in the complete information treatment, which were designed to test their understanding of the instructions. Since the instructions for the limited information case were straightforward, they were not given Review Questions. Afterwards the experimenter checked answers and answered questions. In all complete information sessions the instruction period was within 25 minutes and the entire session took about one hour. In all limited information sessions the instruction period was within 10 minutes and the entire session took approximately 40 minutes. There was no practice round in any session. The average earning was $19.03.

Instructions for the experiments are in Appendix A. Experimental data are available from the authors upon request. Note that in the limited information treatments, players had extremely limited information - they were told that they were in a game, the game length and their strategy space. At the end of each round each player was informed of his own choice in the previous round and his own payoff corresponding to his previous round's choice of quantity. They had no information about the payoff matrix, nor whom they were playing with.

4 Experimental Results

In this section, we compare the performance of the two mechanisms under the complete and limited information conditions. We first examine the level of convergence to Nash equilibrium by checking the proportion of equilibrium play. We then investigate the speed of convergence to Nash equilibrium. Lastly, we examine the efficiency under each treatment.

We follow Chen and Gazzale (2004) for definitions of the level and speed of convergence in an experimental setting. The level of convergence for a round is the proportion of Nash equilibrium play in that round. Similarly, the level of convergence for a block of rounds measures the average proportion of Nash equilibrium play for this block of rounds. The latter smooths out inter-round variation. For example, one can investigate the proportion of Nash equilibrium play in the last 10 rounds in a session with 50 rounds to check the convergence level towards the end of the game. While the speed of convergence should measure how quickly all players converge to equilibrium strategies, in an experimental setting where we do not observe perfect convergence, we adopt a more practical measure. We use the coefficient of $\ln$(Period) to measure the speed of convergence, where Period is the number of rounds a participant has played the game. We will explain this measure in more detail below.

[Figures 1 and 2 about here.]

Figures 1 and 2 present the experimental data under complete and limited information respectively. In each figure, the top four panels present the time series mean strategies (dots), standard deviation (error bars) and equilibrium values (dashed lines) of each of the four types averaged across five independent sessions under ACP. The bottom four panels present the same information under SRL. Note that in the ACP panels, multiple equilibria correspond to two dashed lines representing the upper and lower bound of the equilibrium values. Comparing the top with the bottom four panels, it seems that SRL converges to equilibrium much faster than ACP in both figures. Another important feature is that convergence seems much faster under complete information. In what follows, we will present statistical analysis of these patterns.

We use the proportion of Nash equilibrium play as a measure for the level of convergence. We use the point prediction of $(6, 7, 8, 9)$ for the SRL mechanism, and a set prediction of $(\{3, 4, 5\}, \{9, 10, 11\}, \{13, 14, 15\}, \{15, 16, 17\})$ for the ACP mechanism. Note that the set prediction gives ACP an advantage, since it allows combinations of strategies that are not Nash equilibrium to be counted as equilibrium play.

[Table 4 about here.]

Table 4 tabulates the proportion of Nash equilibrium play over all 50 rounds and in the last 10 rounds in each treatment. We use P^e to denote the proportion of equilibrium play.

RESULT 1 (Equilibrium Play: Comparison of Mechanisms) : *Under complete information, the proportion of Nash equilibrium play under SRL is significantly higher than that under ACP. Under limited information, the proportion of Nash equilibrium play under SRL is weakly higher than that under ACP over all 50 rounds, and significantly higher than that under ACP in the last 10 rounds.*

SUPPORT: Table 4 presents the proportion of Nash equilibrium play for each session. Permutation tests under the null hypothesis that the proportion of Nash equilibrium play under SRL is the same as that under ACP show that, over all rounds, $P^e(SRL_c) > P^e(ACP_c)$ at $p = 0.004$ (one-tailed), while $P^e(SRL_l) > P^e(ACP_l)$ at $p = 0.0833$ (one-tailed). Over the last 10 rounds, we have $P^e(SRL_c) > P^e(ACP_c)$ and $P^e(SRL_l) > P^e(ACP_l)$ at $p < 0.01$ (one-tailed). ■

Result 1 indicates that the proportion of Nash equilibrium play is higher under SRL than that under ACP under both information conditions. This result rejects Hypothesis 1, and it is consistent with Hypothesis 2. Under complete information, even though both mechanisms are dominance solvable and thus we expect their performance to be the same, SRL performs significantly better than ACP. This result is also in contrast to Chen (2003), where she finds that in a two-type environment, the performance of the two mechanisms is statistically indistinguishable under complete information, though SRL performs significantly better than ACP under limited information. Our interpretation is that with two types, the mechanisms can be presented as a bi-matrix game, where it is relatively easy to find Nash equilibrium. With four types, the bi-matrix game representation is no longer feasible. As a result, the uniform dominance (or overwhelming) property of the SRL game helps subjects to get to equilibrium even under complete information.

RESULT 2 (Equilibrium Play: Comparison of Information Conditions) : *For both the SRL and ACP mechanisms, the proportion of equilibrium play under complete information is significantly higher than that under limited information.*

SUPPORT: Table 4 presents the proportion of Nash equilibrium play for each independent observation. Permutation tests show that, over all rounds and the last 10 rounds, $P^e(SRL_c) > P^e(SRL_l)$ at $p = 0.0040$ (one-tailed), while $P^e(ACP_c) > P^e(ACP_l)$ at $p = 0.0040$ (one-tailed). ∎

Result 2 indicates that the amount of information significantly impact the level of convergence, possibly through its influence on the speed of convergence. We are interested in the speed of convergence both at the aggregate mechanism level and at the individual level. The speed of convergence at the individual level might provide an explanation for the dynamics we observe.

[Table 5 about here.]

To investigate the speed of convergence and various factors that affect the speed of convergence, we use a random-effects GLS model, where each group consists of all quantities submitted by one individual. Results of the estimation are reported in Table 5. In six different specifications (columns (1) to (6)), the dependent variable is the distance between actual quantity demanded and equilibrium quantity for the individual player, $|q_i^t - q_i^e|$. Again, we use the equilibrium point prediction for SRL and the set prediction for ACP. In specifications (1) and (3), we use $\ln$(Period) as the independent variable to investigate whether Period (or time) has a significant effect on the speed of convergence. To examine whether the effects of learning remain constant, decreasing or increasing over time, we used Period, $\ln$ (Period), as well as Period^2 as independent variables. Since specifications with $\ln$ (Period) overall yields the best fit, we report only these specifications. In specifications (2) and (4), we add a dummy variable for information conditions, DummyI, which is equal to one for complete information and zero for limited information. The interaction of DummyI and $\ln$(Period) captures the effects of more information on the speed of convergence. In specifications (5) and (6), we add a mechanism dummy, DummyM, which is equal to one for SRL and zero for ACP. Compared with the coefficient of $\ln$(Period), the coefficient for the interaction term, DummyM $\times$ $\ln$(Period), captures the difference between SRL and ACP on the speed of convergence.

RESULT 3 (Speed of Convergence: Information and Mechanism Effects) : *Convergence to equilibrium significantly increases over time. More information significantly increases the speed of convergence for ACP. Under both information conditions, convergence is significantly more rapid under SRL.*

SUPPORT: Table 5 reports results of random-effects GLS regressions. In specifications (1) and (3), the coefficients of $\ln$ (Period) are both negative and highly significant, indicating increased convergence over time. In specifications (2) and (4), the coefficients for DummyI $\times$ $\ln$(Period) are both negative, but only significant under ACP. In specifications (5) and (6), the coefficients for DummyM $\times$ $\ln$(Period) are both negative and highly significant, indicating more rapid convergence under SRL than under ACP. ∎

The first part of Result 3 indicates that players learn to play equilibrium strategies over time, which is not surprising. The second part indicates that more information increases the speed of convergence, but this information effect is only significant for ACP, not for SRL. Unlike ACP, the speed of convergence under SRL does not depend critically on the amount of information players have about the underlying structure of the game. Therefore, if the SRL mechanism is used in limited information settings, such as the Internet, we expect the same speed of convergence as in complete information settings. While the third part of Result 3 is consistent with Hypothesis 2, it rejects Hypothesis 3.

We now examine whether there exist type specific effects on the level and speed of convergence. We use a random-effects GLS model, where the dependent variable is again the distance between actual quantity demanded and equilibrium quantity. In each of the four specifications, the independent variables are the type dummies (Type i Dummy, where $i = 2, 3, 4$), $\ln$(Period), and interactions of type dummies and $\ln$ (Period). The omitted dummy variable is Type 1. Therefore, the Constant measures the level of convergence of Type 1, while the coefficient of Type i Dummy measures the difference in the level of convergence between Type i and Type 1. Similarly, the coefficient of $\ln$(Period) measures the speed of convergence for Type 1, while the coefficient of Type i Dummy $\times$ $\ln$(Period) measures the difference in the speed of convergence between Type i and Type 1. Results of the estimation is reported in Table 6.

[Table 6 about here.]

Under SRL, while the levels of convergence, as captured by the coefficients of type dummies, do not differ significantly across types at the conventional level,[8] the speed of convergence does.

RESULT 4 (Speed of Convergence by Type under SRL) : *Under SRL complete information, the speed of convergence follows the order of Type 1 $\sim$ Type 2 $\sim$ Type 4 $>$ Type 3. Under SRL limited information, the speed of convergence follows the order of Type 1 $>$ Type 2 $>$ Type 3 $\sim$ Type 4.*

SUPPORT: Specification (1) and (2) in Table 6 reports results of estimation under SRL complete and limited information respectively. Coefficient of $\ln$(Period), and coefficient of $\ln$(Period) $+$ coefficient of Type i Dummy $\times$ $\ln$(Period) measures the speed of convergence of Type 1 and Type i respectively. The more negative a coefficient is, the faster the speed of convergence is. ∎

Result 4 reveals how individual learning takes place under SRL and why it converges so robustly under both complete and limited information conditions. Recall that under SRL a player's cost share is independent of demands higher than her own. Therefore, the smallest user solves an individual optimization (or hill-climbing) problem. Once the smallest user finds the equilibrium, the second smallest user's problem also becomes an individual optimization problem rather than a game. And so on. Regardless of information conditions, once users 1 to i finds the equilibrium quantities and settle down, the $(i+1)^{th}$ user's problem becomes very simple. Under complete information, some players might be able to figure out the equilibrium

[8] We use a 5% statistical significance level to claim existence of any causal effects. Note that under SRL, the levels of convergence for types 2 and 3 are weakly higher ($p < 0.10$) than those for types 1 and 4 under complete information.

quantities through various degrees of introspection without waiting for smaller users to settle down first. Therefore, while the order of settling down helps the speed of convergence, it is not crucial. We do observe that Type 1, 2 and Type 4 have statistically indistinguishable speed of convergence. In limited information settings, however, this order of settling down becomes especially important, as rational introspection is not feasible, while experimentation and hill-climbing are the key elements of learning. The second part of Result 4 indicates that the speed of convergence by type follows essentially the same order of settling down analyzed before, with the speed of types 3 and 4 statistically indistinguishable ($p > 0.10$). This result illustrates the importance of going beyond a two-type design.

Under ACP, the type 1 convergence level is significantly lower than the other types under both complete and limited information. This pattern is also confirmed from Figures 1 and 2. The convergence speed, on the other hand, does not exhibit any type-specific pattern as in SRL.

RESULT 5 (Speed of Convergence by Type under ACP) : *Under ACP complete information, the speed of convergence follow the order of Type 1 $>$ Type 4 $>$ Type 3 $>$ Type 2. Under ACP limited information, the speed of convergence is not significantly different across types.*

SUPPORT: Specification (3) and (4) in Table 6 reports results of estimation under ACP complete and limited information respectively. Again, coefficient of $\ln$(Period), and coefficient of $\ln$(Period) + coefficient of Type i Dummy $\times$ $\ln$(Period) measures the speed of convergence of Type 1 and Type i respectively. The more negative a coefficient is, the faster the speed of convergence is. ∎

As discussed in Section 2, under ACP, a player's cost share is affected by everyone else's demand. There does not exist an clear order of settling down by type. With complete information, various degrees of rational introspection might help convergence to equilibrium. Under limited information, however, as one player's experimentation immediately affects every other player's payoff, this makes learning difficult. Result 5 confirms this observation.

Although there is no systematic efficiency comparison between the two mechanisms in general, in this experiment we can make efficiency comparison between the two mechanisms, since we give each player a lump sum payment such that the equilibrium aggregate payoffs for both mechanisms are the same. Group efficiency is calculated by taking the ratio of the sum of the actual earnings of all subjects in a session and the Pareto-optimal earnings of the group without lump-sum payments. Note that in this experimental setting the Pareto optimal payoff without lump sum payments is 881 at strategy four-tuple $(0, 0, 9, 20)$, which is obtained through an exhaustive grid search over the entire strategy space. As a benchmark, the equilibrium aggregate payoff for both mechanisms is 850, which yields an efficiency of 96.48%. We use Ef for efficiency.

RESULT 6 (Efficiency: Comparison of the Two Mechanisms) : *The efficiency of the SRL mechanism is significantly higher than that of the ACP mechanism under both the complete information and the limited information treatments.*

[Table 7 about here.]

SUPPORT: Table 7 reports the efficiency of each session under each treatment. Permutation tests show that

(1) Ef(SRL$_c$) $>$ Ef(ACP$_c$) at a significance level of 0.0040 (one-tailed);

(2) Ef(SRL$_l$) $>$ Ef(ACP$_l$) at a significance level of 0.0040 (one-tailed);

(3) Ef(SRL$_l$) $>$ Ef(ACP$_c$) at a significance level of 0.0040 (one-tailed).

∎

Result 6 says that the SRL mechanism performs robustly better than the ACP mechanism in terms of group efficiency regardless of information conditions. The efficiency of the SRL mechanism under the limited information treatment is significantly higher than the ACP mechanism under the complete information condition. This result is consistent with Result 1 and 3. It is not surprising that it is consistent with Hypothesis 6, but rejects Hypothesis 5. Next, we compare the efficiency within each mechanism under different information conditions.

RESULT 7 (Efficiency: Comparison of Information Conditions) : *For both the SRL and ACP mechanisms, the efficiency under complete information is significantly higher than that under limited information.*

SUPPORT: Table 7 reports the efficiency of each independent observation under each treatment. Permutation tests show

(1) Ef(SRL$_c$) $>$ Ef(SRL$_l$) at a significance level of 0.0040 (one-tailed);

(2) Ef(ACP$_c$) $>$ Ef(ACP$_l$) at a significance level of 0.0476 (one-tailed).

∎

This result says that more information is advantageous for aggregate efficiency. It is consistent with Result 2.

Experimental results indicate that under both complete and limited information settings SRL performs robustly better than ACP in terms of proportion of Nash equilibrium play, speed of convergence and efficiency. The property that a user's cost share is independent of larger users implies an order (sorted by the quantity demanded) of settling down to equilibrium strategies under SRL, which facilitates learning and convergence especially under limited information settings. The uniform dominance property of SRL also implies that sampling and experimentation are much more informative than that under ACP.

5 Simulation Results: Robustness of Experimental Results in More General Environments

In this section we assess the extent to which the experimental results in Sections 4 depend on the linearity of the utility function and the quadratic cost function employed. Following Chen (2003), we consider nine different environments. For simplicity we use polynomial utility and cost functions. The utility function is $\pi_i(x_i, q) = \alpha_i q_i^b - x_i$, where α_i denotes agent i's marginal utility for the output, $b = 0.5, 1$, and 2, and x_i is her cost share. The cost function is chosen to be $C(q) = q^c$, where $c = 0.5, 1$ and 2. Varying parameters b and c will give us nine combinations of concave, linear and convex utility and cost functions. Note that $b = 1$ and $c = 2$ corresponds to the original experimental design.

For the robustness check, it is crucial to use the right learning dynamics. There has been a large literature on learning in games and a growing number of learning algorithms (see Fudenberg and Levine (1998) and Camerer (2003), for surveys). Our interest here is not to compare the performance of various learning models, but rather to look for an algorithm which, when calibrated, closely approximates the observed dynamic paths over fifty rounds. In the following subsections we first report the calibration results using the chosen algorithm. We then report the forecasting results using the calibrated algorithm.

5.1 Calibration

For calibration, we choose to use the payoff assessment learning model, as it is simple, intuitive, and capable of handling both complete and limited information treatments. Furthermore, using the experimental data on cost sharing games reported in Chen (2003), and Van Huyck, Battalio and Rankin's (1996) data on coordination games, Chen and Khoroshilov (2003) show that the payoff-assessment learning model tracks the data the best among three payoff-based learning models: the payoff-assessment learning model (Sarin and Vahid, 1999), a modified experience-weighted attraction learning model (Camerer and Ho, 1999) and a simple reinforcement learning model.

The **payoff-assessment** learning model assumes that a player is a myopic subjective maximizer. She chooses among different strategies only on the basis of the payoff she assesses she would obtain from them. These assessments do not explicitly take into account her subjective judgements regarding the likelihood of alternate states of the world. At each stage, the player chooses the strategy that she myopically assesses to give her the highest payoff and updates her assessment adaptively. Let $u_j(t)$ denote the subjective assessment of strategy s_j at time t, and $\pi_k(t)$ denote the payoff from playing strategy s_k at time t. The initial assessment is denoted by $u_j(0)$. Payoff assessments are updated by taking a weighted average of her previous assessments and the objective payoff she actually obtains at time t. Let r be the discount factor. If strategy k is chosen at time t, then

$$u_j(t+1) = (1-r)u_j(t) + r\pi_k(t), \forall j. \tag{1}$$

Suppose that at time t the decision-maker experiences zero-mean, symmetrically distributed shocks, $Z_j(t)$ to her assessment of the payoff she would receive from choosing strategy s_j, for all s_j. Denote the vector of shocks by $Z = (Z_1, \cdots, Z_{12})$, and their realizations at time t by $z(t) = (z_1(t), \cdots, z_{12}(t))$. The decision maker makes choices on the basis of her shock-distorted subjective assessments, denoted by $\tilde{u}(t) = u(t) + Z(t)$. At time t she chooses strategy s_j if

$$\tilde{u}_j(t) > \tilde{u}_l(t), \forall s_l \neq s_j. \tag{2}$$

Note that mood shocks only affect her choices and not the manner in which assessments are updated. Sarin and Vahid (1999) prove that such a player converges to stochastically choose the strategy that first order stochastically dominates another among the strategies she converges to play with positive probability.

For parameter estimation, we conduct Monte Carlo simulations designed to replicate the characteristics of each of the experimental settings. We then compare the simulated paths with the actual paths of a subset of the experimental data to estimate the parameters which minimize the mean-squared deviation scores.

In each simulation, 10,000 players were created. In each simulation the following steps were taken:

1. Initial values: Since Kolmogorov-Smirnov tests of the round one price distribution by experimental subjects reject the null hypotheses of uniform distribution, we followed the convention in the literature and used the actual first round empirical distribution of choices to generate the first round choices.

2. Simulated players were randomly rematched into groups of four for each period.

3. Shocks are drawn from a uniform distribution, $[-a, a]$.

4. The simulated players' strategies were determined via Eq. (2).

5. Payoffs were determined using the SRL or ACP payoff rule.

6. Assessments were updated according to Eq. (1), using discount factor, r.

[Table 8 about here.]

Table 8 reports the calibrated parameters (discount factor and the interval of mood shocks) for each treatment. Under each mechanism, the interval of mood shocks, $[-a, a]$ are much larger under the limited information treatment than the corresponding complete information treatment, indicating more noisy play under limited information.

5.2 Forecasting

In this subsection, we use the calibrated parameters to simulate the dynamic paths of the two mechanisms in nine different environments. In each environment, we run 10,000 Monte Carlo simulations with the calibrated payoff assessment learning model. Of the nine environments, we report one in each of the three utility functions. Results of the simulation are reported in Figures 3 to 5, which represent three out of the nine simulated environments. The other six figures are not shown here due to space limitations, but are available from the authors upon requests.

[Figures 3 to 5 about here.]

Each figure consists of eight panels. The top four panels report the mean demands (diamonds), standard deviations (error bars) and equilibrium (dark horizontal line) over fifty rounds for types 1 to 4 under ACP, while the bottom four panels report the same information under SRL. We summarize the findings as follows.

RESULT 8 (Simulation) : *With concave and linear utility function, regardless of the form of cost functions, SRL performs better than ACP in terms of the level and speed of convergence. With convex utility functions, regardless of the form of cost functions, the level and speed of convergence are indistinguishable under SRL and ACP.*

SUPPORT: Figures 3 and 4 report the dynamic paths of the two mechanisms with concave and linear utility functions respectively. SRL converges to equilibrium faster and the error bars are smaller than ACP. Figure 5 reports the dynamic paths of the two mechanisms with convex utility function. Speed and level (error bars) of convergence are indistinguishable between the top and lower panels in each figure. ∎

Simulation results indicate that when there are decreasing or constant marginal utility of the quantity demanded, SRL performs robustly better than ACP. When we have increasing returns, the performance of the two mechanisms are similar. In our simulation, cost structure does not seem to affect the ranking of the performance.

6 Conclusion

Cost sharing mechanisms have many practical applications in the real world. An increasingly important area is distributed systems like the Internet, where agents have very limited information about the payoff structure as well as the characteristics of other agents. Most current Internet routers use the FIFO or average cost pricing mechanism, while this study suggests that the fair queuing or serial mechanism might be a better choice.

This paper reports experimental results on the serial and the average cost pricing mechanisms under two different treatments. The first is a complete information treatment designed to test the basic properties of the mechanisms. The other simulates distributed systems by giving the subjects very limited information about the game. The latter present a more challenging and realistic setting for the cost sharing mechanisms.

Experimental results show that the serial mechanism performs significantly better than the average cost pricing mechanism in all treatments both in terms of efficiency and predictability measured as frequency of equilibrium play, as well as the speed of convergence. Simulation results indicate that the experimental results holds when preferences exhibit decreasing or constant returns. The performance of the two mechanisms are similar with increasing returns.

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APPENDIX A. EXPERIMENT INSTRUCTIONS

Instruction for Mechanism S corresponds to the serial mechanism under complete information. Instruction for Mechanism A corresponds to the average cost pricing mechanism under complete information. Introduction, Procedure and Computer instructions for Mechanism A are identical to that of Mechanism S and hence are omitted. Instruction for Mechanism XY is for both mechanisms under limited information. Payoff tables can be found in the first author's web site: http://www.si.umich.edu/˜yanchen/

Experiment Instructions – Mechanism S

Name __________ PCLAB ___ Total Payment _____

Introduction

- You are about to participate in a decision process in which one of numerous alternatives is selected in each of 50 rounds. This is part of a study intended to provide insights into certain features of decision processes. If you follow the instructions carefully and make good decisions you may earn a considerable amount of money. You will be paid in cash at the end of the experiment.
- *During the experiment, we ask that you please do not talk to each other.* If you have a question, please raise your hand and an experimenter will assist you.

Procedure

- At the beginning of the experiment you will be randomly assigned to one of four types: the Blue type, the Green type, the Red type, or the Yellow type. There will be 3 participants of each type. You will keep your type for the entire experiment.
- In each of 50 rounds, you will be randomly matched into different groups. Each group consists of four participants - a Blue, a Green, a Red and a Yellow type. You will not know the identities of the other participants in your group. Your payoff each round depends only on the decisions made by you and the other participants within your group.
- In each of 50 rounds, each participant will demand a quantity, which will give you some benefit. The total quantity within each group will be produced and the cost of production will be shared among all four members of the group. The benefit and cost allocation method will be explained below.

Payoffs

- **Per Round Benefit:** Each unit you demand will give you some benefit.

$$\begin{aligned}
\mathrm{Blue's Benefit} &= (48 \times \mathrm{Blue's Quantity}) + 60 \\
\mathrm{Green's Benefit} &= (54 \times \mathrm{Green's Quantity}) + 20 \\
\mathrm{Red's Benefit} &= 58 \times \mathrm{Red's Quantity} \\
\mathrm{Yellow's Benefit} &= 60 \times \mathrm{Yellow's Quantity}
\end{aligned}$$

- **Per Round Cost:** Your cost share depends on your quantity as well as the quantities demanded by others in your group that are lower than yours. We order the quantities demanded from the lowest to the highest: $Q_1 \leq Q_2 \leq Q_3 \leq Q_4$. The total cost of producing all demanded quantities is the sum of all quantities squared, $(Q_1 + Q_2 + Q_3 + Q_4)^2$. The total cost is distributed to the four participants in the following way.

  - If you demand a quantity which is the smallest in your group, Q_1, your cost share only depends on your own quantity, i.e.,
  $$C_1 = \frac{(4Q_1)^2}{4} = 4Q_1^2.$$
  Therefore, you pay one fourth of the cost of producing four times the smallest quantity.
  - If your demand is Q_2, your cost share is
  $$C_2 = C_1 + \frac{(Q_1 + 3Q_2)^2 - (4Q_1)^2}{3}.$$
  Therefore, you pay the cost share of the smallest demander, plus one third of the additional cost of producing the smallest quantity and three times your own quantity.
  - If your demand is Q_3, your cost share is
  $$C_3 = C_2 + \frac{(Q_1 + Q_2 + 2Q_3)^2 - (Q_1 + 3Q_2)^2}{2}.$$
  Therefore, you pay the cost shares of the second smallest demander, plus half of the additional cost of producing $Q_1 + Q_2 + 2Q_3$.
  - If you demand the highest quantity in your group, Q_4, you pay the rest of the cost:
  $$C_4 = C_3 + [(Q_1 + Q_2 + Q_3 + Q_4)^2 - (Q_1 + Q_2 + 2Q_3)^2].$$

  Therefore, the more you demand, the more cost you have to pay. Your cost share is only affected by your own demand, and those whose demands are lower than yours. Your cost share is independent of demands higher than your own.

- **Per Round Payoff = Per Round Benefit - Per Round Cost**

  Table 1 displays per round payoffs, which summarize both the benefit and the cost, for different types of participants, if that participant's demand is the lowest in his/her group. Payoff tables for participants whose demands are not the smallest are somewhat cumbersome, and thus not displayed.

- There will be 50 rounds. There will be no practice rounds. From the first round, you will be paid for each decision you make.

- Your total payoff is the sum of your payoffs in all rounds.

- The exchange rate is $1 for ____________ points.

Information At the end of each **round**, you are informed of all results for the round:

- The demands of each participant in each group; and

- The corresponding payoffs of each participant in each group.

We encourage you to earn as much cash as you can. Are there any questions?

Review Questions

1. You are a ____ (Blue, Green, Red, Yellow) type.

2. If you demand a quantity of 17 and your demand is the lowest in your group, your payoff will be ____. (Check Table 1.)

3. If the smallest quantity in your group is $Q_1 = 3$, and your demand is the second smallest, $Q_2 = 10$, then
   your benefit = ____;
   your cost = $4Q_1^2 + \frac{(Q_1+3Q_2)^2-(4Q_1)^2}{3}$ =____; and
   your payoff = your benefit - your cost = ____.

4. True or false:

   (a) __You will keep your type for the entire experiment.

   (b) __You will be playing with the same three participants for the entire experiment.

   (c) __Your payoff depends only on your own quantity.

Computer Instructions

Process

- At the beginning of each round, you enter your Quantity, and then click the Okay button to submit it.
- You are free to enter any integer between 0 and 20.
- Notice that if you enter a Quantity outside of 0 and 20, or do not enter an integer, the computer will tell you that your Quantity is not valid and you need to change your selection.
- After all participants have submitted a Quantity, the computer will calculate your payoff and send this number and other relevant information to your screen.
- This process will be repeated for each round.

Changing Your Entry

- Prior to clicking the **Okay** button, use the **Back Space** key to delete your selection, and then enter your new selection.
- *Once you have submitted your Quantity, you cannot change it.*

History Box

- At any point in the experiment, you can review all of your previous choices and payoffs by reviewing the **History** box.
- To view rounds that are not visible, use the scroll bar on the right of the **History** box.

Experiment Instructions – Mechanism A

Name __________ PCLAB ___ Total Payment _____

. . . . . .

Payoffs

- **Per Round Benefit:** Each unit you demand will give you some benefit.

$$\begin{aligned}
\text{Blue's Benefit} &= (48 \times \text{Blue's Quantity}) + 180 \\
\text{Green's Benefit} &= (54 \times \text{Green's Quantity}) + 102 \\
\text{Red's Benefit} &= 58 \times \text{Red's Quantity} \\
\text{Yellow's Benefit} &= 60 \times \text{Yellow's Quantity}
\end{aligned}$$

- **Per Round Cost:** Your cost share depends on your quantity as well as the quantities demanded by others in your group. Cost of producing x units is x^2. Your share of the cost is proportional to your demand. Therefore,

$$\begin{aligned}
\textbf{Your Cost Share} &= \frac{\text{Your Quantity}}{\text{Total Quantity}} \times (\text{Total Quantity})^2 \\
&= (\textbf{Your Quantity}) \times (\textbf{Total Quantity}), where
\end{aligned}$$

Total Quantity = Your Quantity + Sum of Other Three Participants' Quantities.

Therefore, the more you demand, the more cost you have to pay. Your cost share is proportional to your quantity.

- **Per Round Payoff = Per Round Benefit - Per Round Cost**

  Tables 1 - 4 display per round payoffs, which summarize both the benefit and the cost, for each type of participants. The first column is your quantity (from 0 to 20). The first row is the sum of the other three participants' quantities (from 0 to 60, with a step size of 2). The numbers in the table are your payoffs corresponding to each combination of your quantity and the sum of others' quantities.

- There will be 50 rounds. There will be no practice rounds. From the first round, you will be paid for each decision you make.

- Your total payoff is the sum of your payoffs in all rounds.

- The exchange rate is $1 for ____________ points.

Information At the end of each **round**, you are informed of all results for the round:

- The demands of each participant in each group; and

- The corresponding payoffs of each participant in each group.

We encourage you to earn as much cash as you can. Are there any questions?

Review Questions

1. You are a ____ (Blue, Green, Red, Yellow) type.

2. If you demand a quantity of 17 and the sum of the others quantities is 20, your payoff will be ____. (Check Tables 1 - 4, ONE of which is your payoff table.)

3. True or false:

   (a) __You will keep your type for the entire experiment.

   (b) __You will be playing with the same three participants for the entire experiment.

   (c) __Your payoff depends only on your own quantity.

Experiment Instructions – Mechanism XY

PCLAB __ Total Payment ______

Procedure

- You are part of a game, in which you have to make a decision in each of 50 rounds.
- In each round, you are free to enter any integer between 0 and 20.

Information

- At the end of **each round**, you are informed of your result for the round:

- your own choice
- your own payoff

Total Payoffs

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is $1 for ___ points.

We encourage you to earn as much cash as you can. Are there any questions?

Computer Instructions

Process

- At the beginning of each round, you enter your Choice, and then click the Okay button to submit it.
- You are free to enter any integer between 0 and 20.
- Notice that if you enter a Choice outside of 0 and 20, or do not enter an integer, the computer will tell you that your Choice is not valid and you need to change your selection.
- After all participants have submitted a Choice, the computer will calculate your payoff and send this number and other relevant information to your screen.
- This process will be repeated for each round.

Changing Your Entry

- Prior to clicking the **Okay** button, use the **Back Space** key to delete your selection, and then enter your new selection.
- *Once you have submitted your Choice, you cannot change it.*

History Box

- At any point in the experiment, you can review all of your previous choices and payoffs by reviewing the **History** box.
- To view rounds that are not visible, use the scroll bar on the right of the **History** box.

APPENDIX B. Discretization and Multiple Equilibria in ACP

Proof of Proposition 1: Let $\{q_i^*\}_i$ be the Nash equilibrium quantities of the ACP game with a continuous strategy space. With a quadratic cost function, $C(\sum_i q_i)$, the unique Nash equilibrium is characterized by the solution to the following maximization problem:

$$\max_{q_i} \alpha_i q_i - \frac{q_i}{\sum_j q_j}(\sum_j q_j)^2.$$

The first order condition is $\alpha_i - \sum_j q_j - q_i = 0$. Summing over i, we get $\sum_i q_i = \sum_i \alpha_i/(n+1)$. Therefore,

$$q_i^* = \alpha_i - \frac{\sum_j \alpha_j}{n+1}, \ \sum_i q_i^* = \frac{\sum_i \alpha_i}{n+1}, \text{ and } \pi_i^* = (q_i^*)^2.$$

To prove that $\{\bar{q}_1, \cdots, \bar{q}_n | \bar{q}_i \in \{q_i^* - s, q_i^*, q_i^* + s\}$ and $\sum_i \bar{q}_i = \sum_i q_i^*\}$ are all Nash equilibria of the discrete game, we need to show that unilateral defection by any player does not improve her payoff. In equilibrium

$$\bar{\pi}_i(\bar{q}) = \alpha_i \bar{q}_i - \bar{q}_i \sum_j \bar{q}_j = \alpha_i \bar{q}_i - \bar{q}_i \sum_j q_j^* = \bar{q}_i(\alpha_i - \frac{\sum_i \alpha_i}{n+1}) = \bar{q}_i q_i^*.$$

Case 1. $\bar{q}_i = q_i^* - s$. In this case $\bar{\pi}_i(\bar{q}) = (q_i^* - s)q_i^*$.

If player i unilaterally defects to strategy $q_i = q_i^* - m \equiv \bar{q}_i + s - m$, where $m \in D$ and $m \neq s$, $\pi_i(q_i, \bar{q}_{-i}) = (q_i^* - m)[\alpha_i - (\sum_j q_j^* + s - m)] = (q_i^* - m)(q_i^* + m - s) = (q_i^* - s)q_i^* - m(m-s) \leq (q_i^* - s)q_i^*$, since $m(m-s) \geq 0$ for $m \in D$.

Case 2. $\bar{q}_i = q_i^*$. In this case $\bar{\pi}_i(\bar{q}) = (q_i^*)^2$.

If player i unilaterally defects to strategy $q_i = q_i^* + m \equiv \bar{q}_i + m$, where $m \in D$ and $m \neq 0$, $\pi_i(q_i, \bar{q}_{-i}) = (q_i^* + m)[\alpha_i - (\sum_j q_j^* + m)] = (q_i^* + m)(q_i^* - m) = (q_i^*)^2 - m^2 < (q_i^*)^2$.

Case 3. $\bar{q}_i = q_i^* + s$. In this case $\bar{\pi}_i(\bar{q}) = (q_i^* + s)q_i^*$.

If player i unilaterally defects to strategy $q_i = q_i^* + m \equiv \bar{q}_i - s + m$, where $m \in D$ and $m \neq s$, $\pi_i(q_i, \bar{q}_{-i}) = (q_i^* + m)[\alpha_i - (\sum_j q_j^* - s + m)] = (q_i^* + m)(q_i^* - m + s) = (q_i^* + s)q_i^* - m(m-s) \leq (q_i^* + s)q_i^*$, since $m(m-s) \geq 0$ for $m \in D$.

Therefore, $\{\bar{q}_1, \cdots, \bar{q}_n | \bar{q}_i \in \{q_i^* - s, q_i^*, q_i^* + s\}$ and $\sum_i \bar{q}_i = \sum_i q_i^*\}$ are all Nash equilibria of the discrete game.

Let $\bar{q}_1 \leq \bar{q}_2 \leq \cdots \leq \bar{q}_n$. Let $\bar{q}_i = q_i^* + s_i$, where $s_i = -s, 0$ or s and $\sum_i s_i = 0$. The aggregate payoffs in equilibrium is

$\sum_i \pi_i(\bar{q}_i) = \sum_i \bar{q}_i q_i^* = \sum_i (q_i^*)^2 + \sum_i s_i q_i^*$. Q.E.D.

Subjects		Parameters			Equil. Quantities				Equil. Payoffs	
ID	Label	α_i	ω_i^s	ω_i^a	q_i^s	q_i^a			π_i^s	π_i^a
1	(Blue)	48	60	180	**6**	$\{3,$	**4**,	$5\}$	204	**196**
2	(Green)	54	20	102	**7**	$\{9,$	**10**,	$11\}$	203	**202**
3	(Red)	58	0	0	**8**	$\{13,$	**14**,	$15\}$	213	**196**
4	(Yellow)	60	0	0	**9**	$\{15,$	**16**,	$17\}$	230	**256**
	Total	220	80	282	30		44		850	**850**

Table 1: Parameters, Equilibrium Quantities and Payoffs. Note: bold-faced quantities and payoffs are Nash equilibrium quantities and payoffs with a continuous strategy space. For ACP all Nash equilibrium quantities add up to 44.

Number	q_1^a	q_2^a	q_3^a	q_4^a	π_1^a	π_2^a	π_3^a	π_4^a	$\sum_i \pi_i^a$
1	3	9	15	17	192	192	210	272	866
2	3	10	14	17	192	202	196	272	862
3	3	10	15	16	192	202	210	256	860
4	3	11	13	17	192	212	182	272	858
5	3	11	14	16	192	212	196	256	856
6	3	11	15	15	192	212	210	240	854
7	4	9	14	17	196	192	196	272	856
8	4	9	15	16	196	192	210	256	854
9	4	10	13	17	196	202	182	272	852
10	**4**	**10**	**14**	**16**	**196**	**202**	**196**	**256**	**850**
11	4	10	15	15	196	202	210	240	848
12	4	11	13	16	196	212	182	256	846
13	4	11	14	15	196	212	196	240	844
14	5	9	13	17	200	192	182	272	846
15	5	9	14	16	200	192	196	256	844
16	5	9	15	15	200	192	210	240	842
17	5	10	13	16	200	202	182	256	840
18	5	10	14	15	200	202	196	240	838
19	5	11	13	15	200	212	182	240	834

Table 2: Multiple Equilibrium Quantities and Payoffs in ACP.

	Information Conditions	
	Complete Information	Limited Information
SRL	SRL_c (5 sessions)	SRL_l (5 sessions)
ACP	ACP_c (5 sessions)	ACP_l (5 sessions)

Table 3: Features of Experimental Treatments

Rounds	All 50 Rounds				Last 10 Rounds			
Information	Complete		Limited		Complete		Limited	
Session	SRL_c	ACP_c	SRL_l	ACP_l	SRL_c	ACP_c	SRL_l	ACP_l
1	0.629	0.215	0.272	0.145	0.935	0.342	0.500	0.183
2	0.418	0.228	0.180	0.160	0.717	0.233	0.342	0.192
3	0.405	0.263	0.137	0.162	0.700	0.342	0.208	0.167
4	0.440	0.197	0.193	0.140	0.525	0.192	0.258	0.125
5	0.583	0.265	0.175	0.183	0.825	0.383	0.292	0.233

Table 4: Proportion of Equilibrium Play for Each Session

	Dependent Variable: Distance between actual quantity and equilibrium quantity					
	SRL		ACP		Complete Info.	Limited Info.
	(1)	(2)	(3)	(4)	(5)	(6)
ln(Period)	-1.0376	-1.0225	-0.6170	-0.5115	-0.4997	-0.7581
	(0.0357)***	(0.0429)***	(0.0457)***	(0.0552)***	(0.0395)***	(0.0570)***
DummyI×ln(Period)		-0.0301		-0.2110		
		(0.0468)		(0.0619)***		
DummyM×ln(Period)					-0.2726	-0.5195
					(0.0469)***	(0.0604)***
Constant	5.1161	5.1159	5.2243	5.2243	3.8278	6.5111
	(0.1650)***	(0.1444)***	(0.1953)***	(0.1894)***	(0.1433)***	(0.1886)***
Observations	5988	5988	6000	6000	5988	6000
Number of groups	120	120	120	120	120	120

Notes:

1. Random-effects GLS regressions.
2. Standard errors in parentheses.
3. DummyI is a dummy variable for the information conditions, while DummyM is a dummy variable for the mechanisms.
4. Significant at: *** 1% level.

Table 5: Speed of Convergence

	Dependent Variable: Distance between actual quantity and equilibrium quantity			
	SRL		ACP	
	Complete Info.	Limited Info.	Complete Info.	Limited Info.
	(1)	(2)	(3)	(4)
Type 2 Dummy	-0.6087	-0.6163	-5.1958	-2.7872
	(0.3610)*	(0.6930)	(0.6561)***	(0.7674)***
Type 3 Dummy	-0.6884	-0.8470	-3.3710	-1.9269
	(0.3610)*	(0.6930)	(0.6561)***	(0.7674)**
Type 4 Dummy	0.4089	-0.1585	-1.7881	-1.2928
	(0.3610)	(0.6930)	(0.6561)***	(0.7674)*
ln(Period)	-0.7159	-1.7802	-1.0769	-0.7840
	(0.0687)***	(0.1229)***	(0.1057)***	(0.1485)***
Type 2 Dummy × ln(Period)	0.1309	0.3181	0.9876	0.3991
	(0.0971)	(0.1738)*	(0.1495)***	(0.2101)*
Type 3 Dummy×ln(Period)	0.2606	0.5355	0.5018	0.3369
	(0.0971)***	(0.1738)***	(0.1495)***	(0.2101)
Type 4 Dummy×ln(Period)	-0.0654	0.5117	0.2703	0.0121
	(0.0971)	(0.1738)***	(0.1495)*	(0.2101)
Constant	3.1330	7.7223	7.3319	7.2071
	(0.2553)***	(0.4900)***	(0.4640)***	(0.5426)***
Observations	2988	3000	3000	3000
Number of groups	60	60	60	60

Notes:

1. Random-effects GLS regressions.
2. Standard errors in parentheses.
3. Significant at: * 10% level; ** 5% level; *** 1% level.

Table 6: Speed of Convergence by Type

Information	Complete		Limited	
Session	SRL_c	ACP_c	SRL_l	ACP_l
1	0.850	0.584	0.767	0.527
2	0.861	0.614	0.733	0.628
3	0.852	0.613	0.736	0.620
4	0.840	0.636	0.769	0.506
5	0.871	0.662	0.763	0.492

Table 7: Efficiency of Each Session

	Session	ACP_c	ACP_l	SRL_c	SRL_l
Session Level MSD	1	0.92	0.93	0.79	0.87
	2	0.93	0.92	0.75	0.90
	3	0.92	0.92	0.80	0.90
	4	0.92	0.93	0.80	0.89
	5	0.92	0.92	0.75	0.90
Overall MSD		0.92	0.93	0.79	0.90
Estimated Parameters	a	2	10	1	10
	r	0.90	0.90	0.70	0.90

Table 8: Calibration of the Payoff Assessment Model

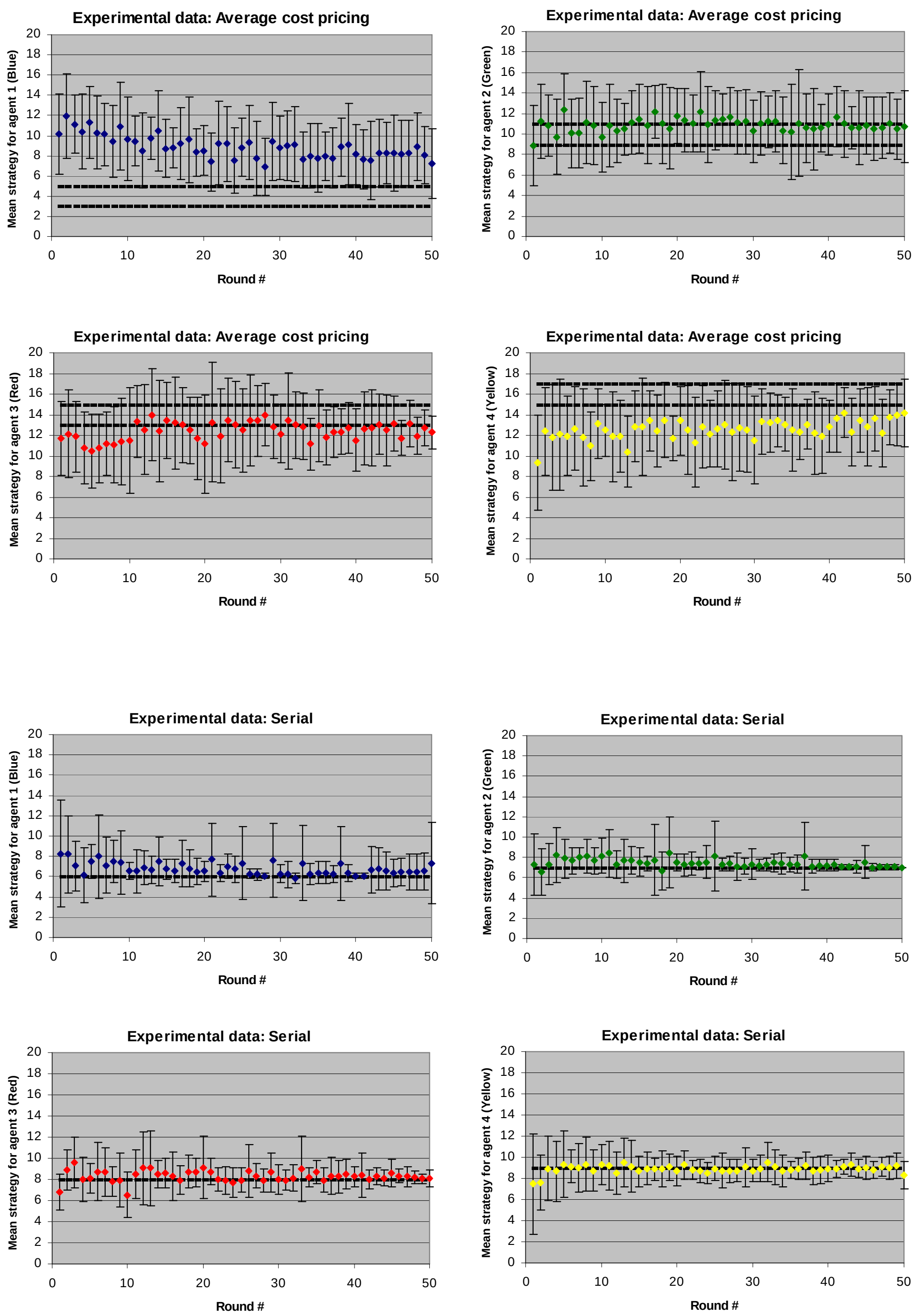

Figure 1: Experimental Data: Complete Information

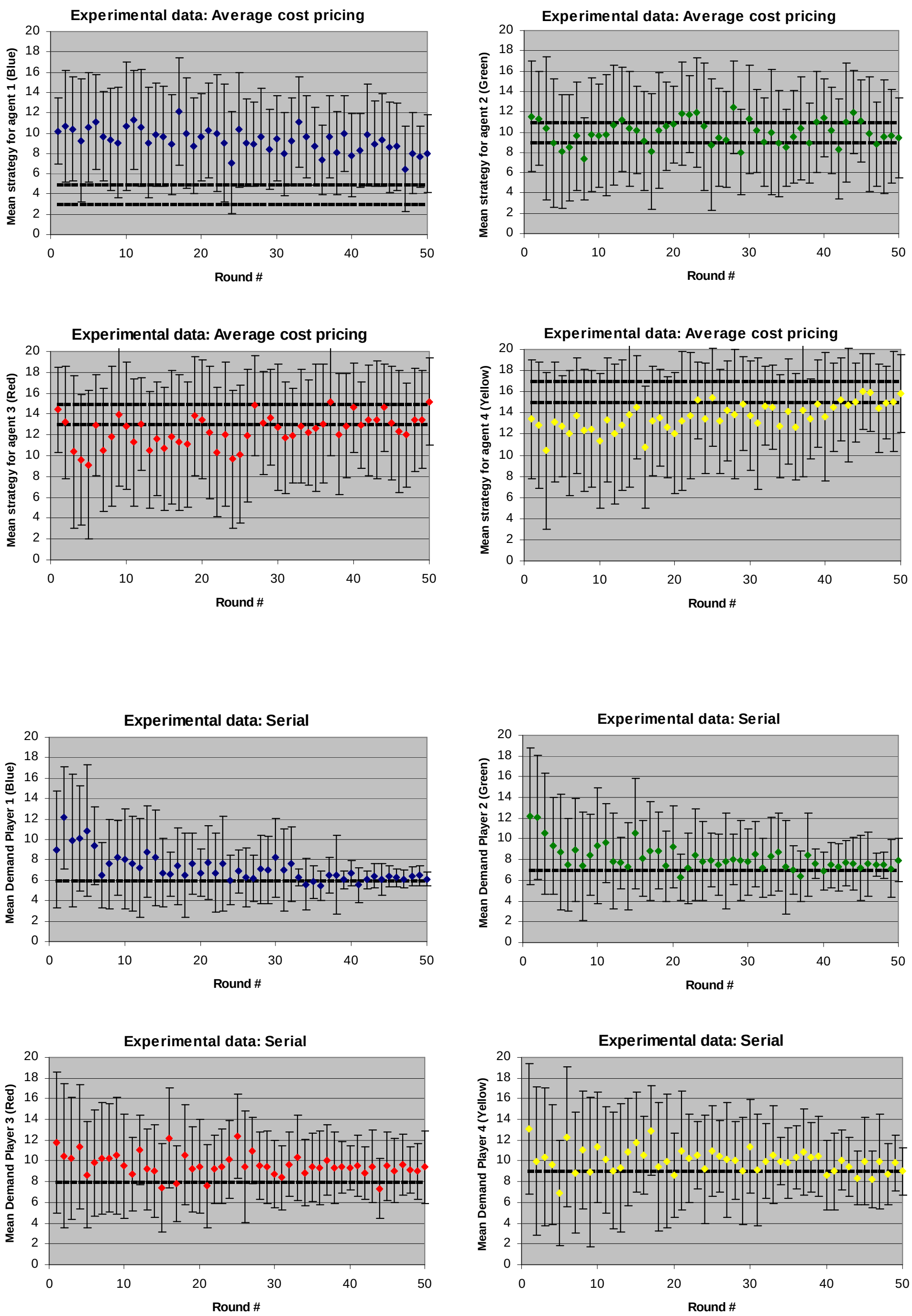

Figure 2: Experimental Data: Limited Information

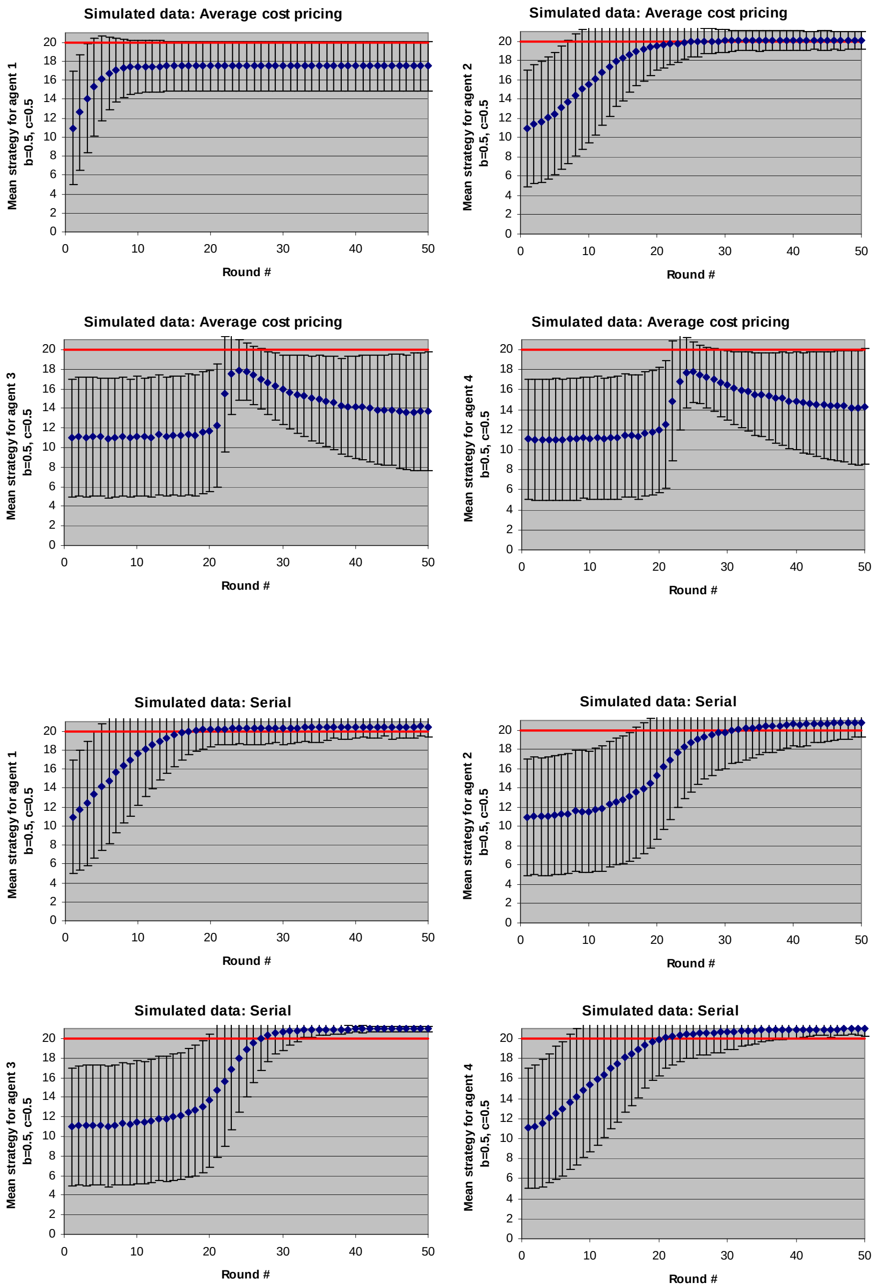

Figure 3: Simulation: b=0.5, c=0.5

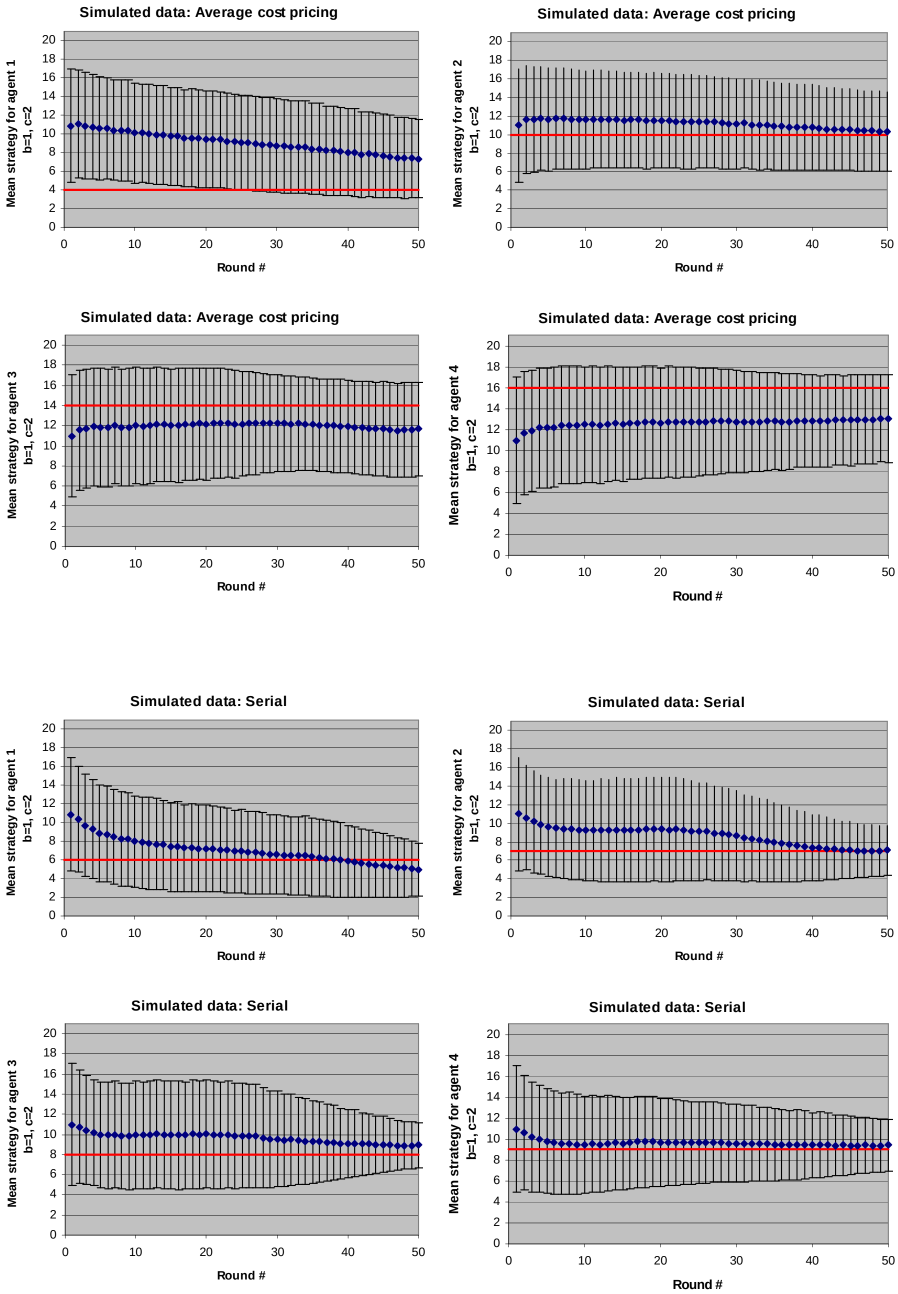

Figure 4: Simulation: b=1.0, c=2.0

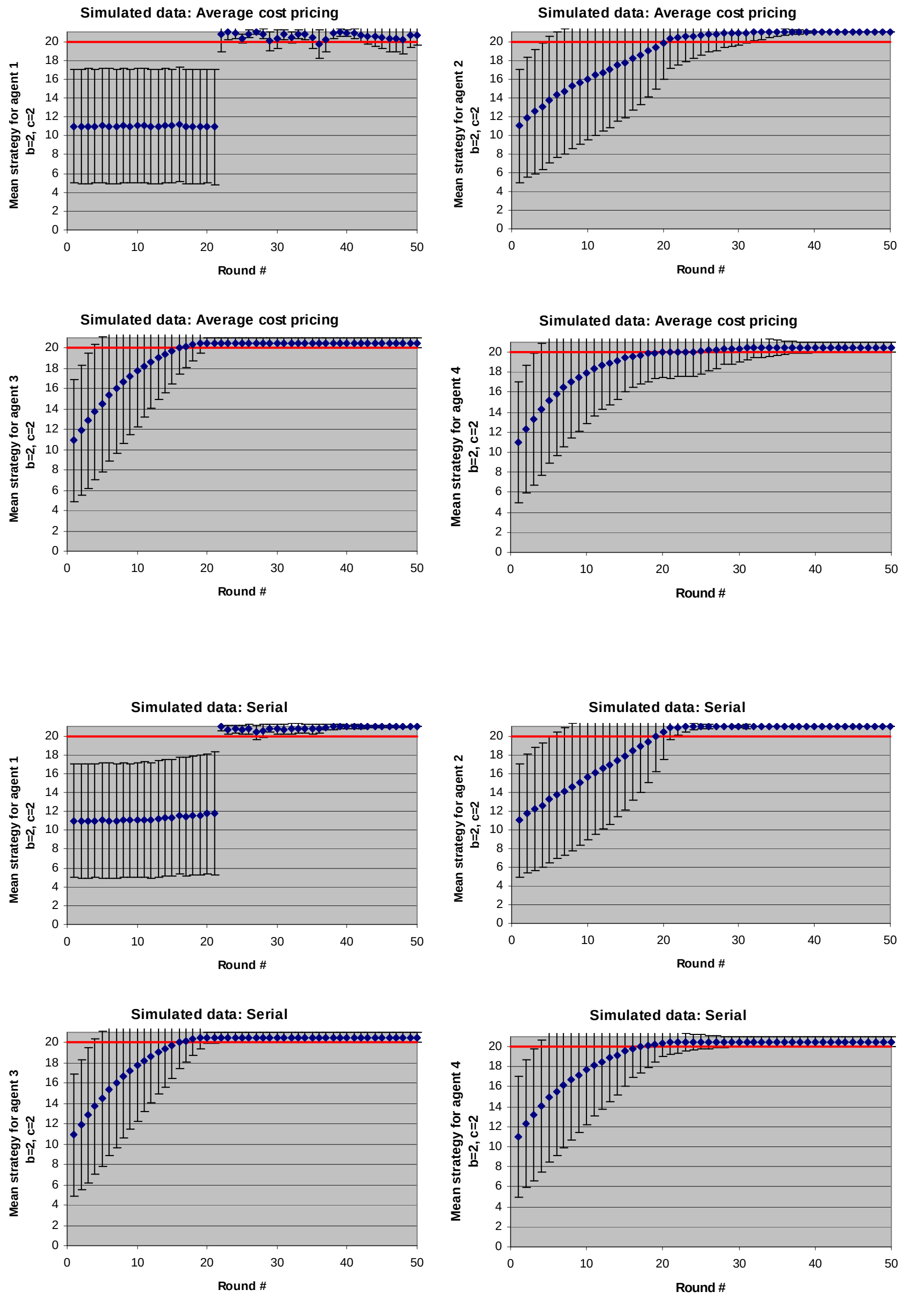

Figure 5: Simulation: b=2.0, c=2.0