## Online Appendix for

## Information Acquisition and Provision in School Choice

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## Appendix Online.A Analyses of the Game in the Experiment with Risk-Neutral Students

Given the payoff table introduced in Section 4, this appendix derives in details the equilibrium strategies and payoffs under the assumption that every student is risk neutral. We also vary information structure and derive the incentive to acquire information. The results on risk-averse students are presented in Appendix Online.B. Throughout, students do not know the realization of tie breakers when playing the game.

### **Online.A.1** Information Structure

We consider the following 5 scenarios where the information structure differs:

(1) Complete information on preferences: Everyone knows her own and others' realized preferences;

(2) Incomplete information on preferences: Everyone knows her own realized preferences but only the distribution of others';

(3) Unknown preferences: Everyone only knows the distribution of her own preferences and of others';

(4) Unknown preferences (Scenario (3)) with acquisition of information on one's own preferences;

(5) Incomplete information (Scenario (2)) with acquisition of information on others' preferences.

The literature on school choice, or on matching in general, focuses on the first two scenarios – complete or incomplete information. By introducing scenarios (3)-(5), we extend the literature by endogenizing the acquisition of information on one's own or on others' preferences.

Figure 1 shows the relationship among the five scenarios.

### **Online.A.2** Scenario (1): Complete Information on Preferences

**The Immediate Acceptance Mechanism** Given any realization of the preferences, we have the following symmetric equilibrium strategies and payoffs under the Immediate Acceptance mechanism (Table 10).

Scenario (1): Complete information on Preferences

Preference realizations are common knowledge

Scenario (5) Acquire information on others' preferences

Scenario (2): Incomplete information on Preferences Preference realizations are private information. Distribution of preferences is common knowledge.

**Scenario (4)** Acquire information on own preferences

Scenario (3): Unknown Preferences Preference realizations are unknown. Distribution of preferences is common knowledge.

#### Figure 1: Scenarios Considered and the Corresponding Information Structure

Realization of	Probability	Action	given realized type	Payoff giver	n realized type
Preference	Realized	(1, 0.1, 0)	(1, 1.1, 0)	(1, 0.1, 0)	(1, 1.1, 0)
(1, 0.1, 0) (1, 0.1, 0) (1, 0.1, 0)	64/125	(a, b, c)	-	11/30	-
(1, 1.1, 0) (1, 0.1, 0) (1, 0.1, 0)	48/125	(a, b, c)	(b, a, c)	1/2	11/10
(1, 1.1, 0) (1, 1.1, 0) (1, 0.1, 0)	12/125	(a, b, c)	(b, a, c)	1	11/20
(1, 1.1, 0) (1, 1.1, 0) (1, 1.1, 0)	1/125	-	(a, b, c) w/ prob. 3/7 <sup>a</sup> (b, a, c) w/ prob. 4/7 <sup>a</sup>	-	7/10

Table 10: Symmetric Equilibrium under IA given Each Realization of Preference Profiles

a. We may allow one student to play (a,b,c) and the other two to play (b,a,c), which is a pure-strategy Nash equilibrium. As long as everyone has the same probability to play (a,b,c), the expected payoff of everyone is also 7/10.

Ex ante, before the realization of the preferences, given that they know they will play the game

with complete information under IA, the expected payoff of each student is:

$$\frac{4}{5}\left(\frac{11}{30}\frac{16}{25} + \frac{1}{2}\frac{8}{25} + \frac{1}{25}\right) + \frac{1}{5}\left(\frac{11}{10}\frac{16}{25} + \frac{11}{20}\frac{8}{25} + \left(\frac{7}{10}\right)\frac{1}{25}\right) = \frac{4}{5}\frac{326}{750} + \frac{1}{5}\frac{681}{750} = \frac{397}{750}.$$

**The DA Mechanism** Before looking at equilibrium, we use the following table to clarify the assignment probabilities given students' actions (Table 11). Note that we always use DA with single tie-breaking.

	Probability of Being Assigned to Each School if							
Submitted	Play	ing (a,	b, c)	Playing (b, a, c)				
List	a	b	с	a	b	с		
(a, b, c)								
(a, b, c)	1/3	1/3	1/3	-	-	-		
(a, b, c)								
(b, a, c)								
(a, b, c)	1/2	1/6	1/3	0	2/3	1/3		
(a, b, c)						·		
(b, a, c)								
(b, a, c)	2/3	0	1/3	1/6	1/2	1/3		
(a, b, c)						,		
(b, a, c)								
(b, a, c)	-	-	-	1/3	1/3	1/3		
(b, a, c)				,	,	•		

Table 11: Assignment Probability under DA given Each Strategy Profile

Given any realization of the preferences, we have the following equilibrium strategies and payoffs under DA (Table 12).

*Ex ante*, before the realization of the preferences, given that they know they will play the game with complete information under DA, the expected payoff to each student is:

$$\frac{4}{5}\left(\frac{11}{30}\frac{16}{25} + \frac{31}{60}\frac{8}{25} + \frac{2}{3}\frac{1}{25}\right) + \frac{1}{5}\left(\frac{22}{30} \times \frac{16}{25} + \frac{43}{60}\frac{8}{25} + \left(\frac{21}{30}\right)\frac{1}{25}\right) = \frac{365}{750}$$

#### **Online.A.3** Scenario (2): Incomplete Information on Preferences

**The Immediate Acceptance Mechanism** When one's own preferences are private information and the distribution of preferences is common knowledge, there is a unique symmetric equilibrium under IA:

$$\sigma_{IA}^{(2)}\left((1,1.1,0)\right) = (b,a,c); \sigma_{IA}^{(2)}\left((1,0.1,0)\right) = (a,b,c)$$

For any given student, there are three possibilities of opponents' types:

For a type-(1, 0.1, 0) student, it is a dominant strategy to play (a, b, c). Conditional on her type,

	1	0			
Realization of	Probability	Action given	n realized type	Payoff given	realized type
Preference	Realized	(1, 0.1, 0)	(1, 1.1, 0)	(1, 0.1, 0)	(1, 1.1, 0)
(1, 0.1, 0) (1, 0.1, 0) (1, 0.1, 0)	64/125	(a, b, c)	-	11/30	-
(1, 1.1, 0) (1, 0.1, 0) (1, 0.1, 0)	48/125	(a, b, c)	(b, a, c)	$ \begin{array}{c} \frac{1}{2} + \frac{1}{60} \\ = 31/60 \end{array} $	$= \frac{2}{3} \frac{11}{10} \\ = \frac{22}{30}$
(1, 1.1, 0) (1, 1.1, 0) (1, 0.1, 0)	12/125	(a, b, c)	(b, a, c)	2/3	$\frac{\frac{11}{20} + \frac{1}{6}}{= 43/60}$
(1, 1.1, 0) (1, 1.1, 0) (1, 1.1, 0)	1/125	-	(b, a, c)	-	21/30

Table 12: Equilibrium under DA given each Realization of Preference Profiles

Types	Probability	Others' Action Profile
(1, 0.1, 0) (1, 0.1, 0)	16/25	(a, b, c) (a, b, c)
(1, 1.1, 0) (1, 0.1, 0)	8/25	(b, a, c) (a, b, c)
(1, 1.1, 0) (1, 1.1, 0)	1/25	(b, a, c) (b, a, c)

her equilibrium payoff is:

$$\frac{16}{25}\left(\frac{1}{3}\left(1+\frac{1}{10}+0\right)\right) + \frac{8}{25}\frac{1}{2} + \frac{1}{25} = \frac{326}{750}.$$

If a type-(1, 0.1, 0) student deviates to (b, a, c), she obtains:

$$\frac{16}{25}\left(\frac{1}{10}\right) + \frac{8}{25}\left(\frac{1}{2}\left(\frac{1}{10}+0\right)\right) + \frac{1}{25}\left(\frac{11}{30}\right) = \frac{71}{750}.$$

For a type-(1, 1.1, 0) student, given others follow  $\sigma_{BM}^{(2)}$ , playing (b, a, c) results in a payoff of:

$$\frac{16}{25}\frac{11}{10} + \frac{8}{25}\left(\frac{1}{2}\left(\frac{11}{10} + 0\right)\right) + \frac{1}{25}\left(\frac{1}{3}\left(\frac{11}{10} + 1 + 0\right)\right) = \frac{681}{750}.$$

If a type-(1, 1.1, 0) student deviates to (a, b, c), she obtains:

$$\frac{16}{25}\left(\frac{1}{3}\left(\frac{11}{10}+1+0\right)\right) + \frac{8}{25}\left(\frac{1}{2}\left(1+0\right)\right) + \frac{1}{25}\left(1\right) = \frac{486}{750}.$$

It is therefore not a profitable deviation. Furthermore, she has no incentive to deviate to other rankings such as (c, a, b) or (c, b, a).

Before the realization of their own preferences while knowing that they will play the game under DA with incomplete information, the *ex ante* payoff to every student is:

$$\frac{326}{750}\frac{4}{5} + \frac{681}{750}\frac{1}{5} = \frac{397}{750}$$

**Remark 1.** Note that the two scenarios, (1) and (2), result in the same ex ante payoffs under IA.

**Remark 2.** In neither scenarios, a type-(1, 0.1, 0) student is ever matched with school b as long as there is at least one type-(1, 1.1, 0) student.

**The DA Mechanism** When one's own preferences are private information and the distribution of preferences is common knowledge, there is a unique equilibrium under DA:

$$\sigma_{DA}^{(2)}\left((1,1.1,0)\right) = (b,a,c); \sigma_{DA}^{(2)}\left((1,0.1,0)\right) = (a,b,c).$$

For any given student, there are three possibilities of opponents' types: For a type-(1, 0.1, 0)

	Types	Probability	Others' Action Profile
1	(1, 0.1, 0) (1, 0.1, 0)	16/25	(a, b, c) (a, b, c)
2	(1, 1.1, 0) (1, 0.1, 0)	8/25	(b, a, c) (a, b, c)
3	(1, 1.1, 0) (1, 1.1, 0)	1/25	(b, a, c) (b, a, c)

student, it is a dominant strategy to play (a, b, c). Conditional on her type, her equilibrium payoff is:

$$\frac{16}{25}\left(\frac{1}{3}\left(1+\frac{1}{10}+0\right)\right) + \frac{8}{25}\left(\frac{1}{2}+\frac{1}{60}+0\right) + \frac{1}{25}\left(\frac{2}{3}+0\right) = \frac{320}{750}.$$

If a type-(1, 0.1, 0) student deviates to (b, a, c), she obtains:

$$\frac{16}{25}\left(\frac{2}{30}+0\right) + \frac{8}{25}\left(\frac{1}{20}+\frac{1}{6}+0\right) + \frac{1}{25}\left(\frac{1}{3}\left(1+\frac{1}{10}+0\right)\right) = \frac{95}{750}.$$

For a type-(1, 1.1, 0) student, given others follow  $\sigma_{DA}^{(2)}$ , playing (b, a, c) results in a payoff of:

$$\frac{16}{25}\left(\frac{2}{3}\frac{11}{10}\right) + \frac{8}{25}\left(\frac{1}{2}\frac{11}{10} + \frac{1}{6}\right) + \frac{1}{25}\left(\frac{1}{3}\left(\frac{11}{10} + 1 + 0\right)\right) = \frac{545}{750}$$

If a type-(1, 1.1, 0) student deviates to (a, b, c), she obtains:

$$\frac{16}{25}\left(\frac{1}{3}\left(\frac{11}{10}+1+0\right)\right) + \frac{8}{25}\left(\frac{1}{2}+\frac{11}{60}\right) + \frac{1}{25}\left(\frac{2}{3}\right) = \frac{520}{750}.$$

It is therefore not a profitable deviation. Furthermore, she has no incentive to deviate to other rankings such as (c, a, b) or (c, b, a).

The ex ante payoff to every student, before knowing their own true preferences, is:

$$\frac{320}{750}\frac{4}{5} + \frac{545}{750}\frac{1}{5} = \frac{365}{750}.$$

**Remark 3.** Note that the two scenarios, (1) and (2), result in the same ex ante payoffs under DA.

**Remark 4.** In both scenarios, there is a positive probability that a type-(1, 0.1, 0) student is matched with school b when there is at least one type-(1, 1.1, 0) student.

#### **Online.A.4** Scenario (3): Unknown Preferences

The Immediate Acceptance Mechanism Under IA, the unique symmetric equilibrium is that everyone plays  $\sigma_{IA}^{(3)} = (a, b, c)$ . The expected payoff of this strategy is:

$$\frac{1}{3}\left(1 + \left(\frac{1}{5}\frac{11}{10} + \frac{4}{5}\frac{1}{10}\right) + 0\right) = \frac{13}{30} = \frac{325}{750}.$$

If an student deviates to (b, a, c), her payoff is:

$$\left(\frac{1}{5}\frac{11}{10} + \frac{4}{5}\frac{1}{10}\right) = 0.3 = \frac{225}{750}.$$

**Remark 5.** In Scenario (2), the ex ante payoff is  $\frac{397}{750}$  which is higher than that of Scenario (3),  $\frac{225}{750}$ .

**Remark 6.** Comparing Scenarios (1), (2), and (3), we can improve the social welfare by making it easier for students to learn their preferences and then transforming (3) into (2) or (1) under the Immediate Acceptance.

**The DA Mechanism** The unique symmetric equilibrium under DA is that everyone plays  $\sigma_{DA}^{(3)} = (a, b, c)$ .

The expected payoff of this strategy is:

$$\frac{1}{3}\left(1 + \left(\frac{1}{5}\frac{11}{10} + \frac{4}{5}\frac{1}{10}\right) + 0\right) = \frac{13}{30} = \frac{325}{750}$$

If an student deviates to (b, a, c), her payoff is:

$$\frac{1}{5}\left(\frac{2}{3}\frac{11}{10}\right) + \frac{4}{5}\left(\frac{2}{3}\frac{1}{10}\right) = 0.2 = \frac{150}{750}.$$

**Remark 7.** In Scenario (2), the ex ante payoff is  $\frac{365}{750}$  which is higher than that of Scenario (3),  $\frac{325}{750}$ .

**Remark 8.** Comparing Scenarios (1), (2), and (3), we can improve the social welfare by making it easier for students to learn their preferences and then transforming (3) into (2) or (1) under DA.

**Remark 9.** The benefit of providing free information on own preferences is higher under the Immediate Acceptance.

**Remark 10.** In Scenarios (3), the Immediate Acceptance mechanism achieves the same outcome as DA.

In the following, we discuss students' incentives to acquire information on one's own preferences.

#### **Online.A.5** Scenario (4): (3) + acquisition of information on one's own preferences

**The Immediate Acceptance Mechanism** Now suppose that students know only the distribution of their own and others' preferences. We consider their incentives to acquire information on their own preferences.

After acquiring the information, both informed and uninformed students know how many others are informed. However, informed students know their own preferences, while uninformed students only know the distribution of own preference.

Willingness to pay for information on own preferences can be defined in the following three cases:

- $w_0^{own}$ : when no other informed students;
- $w_1^{own}$ : when there is another informed student;
- $w_2^{own}$ : when there are two other informed students.

The following table summarizes the equilibrium strategies and *ex ante* payoffs for informed and uninformed players (Table 13).

# of	Players	Strategy:	Strategy:	Informed	Ex An	te Payoff	Willingness to
Informed	Uninformed	Uninformed	(1, 0.1, 0)	(1, 1.1, 0)	Informed	Uninformed	pay for info
0	3	(a, b, c)	-	-	-	$\frac{325}{750}$	$\frac{60}{750}$
1	2	(a, b, c)	(a, b, c)	(b, a, c)	$\frac{385}{750}$	$\frac{335}{750}$	$\frac{49.5}{750}$
2	1	(a, b, c)	(a, b, c)	(b, a, c)	$\frac{384.5}{750}$	$\frac{358}{750}$	$\frac{39}{750}$
3	0	-	(a, b, c)	(b, a, c)	$\frac{397}{750}$	-	

Table 13: Willingness to Pay for Information on Own Payoffs under IA

**Overt and covert information acquisition:** In the current setting, we focus on overt information acquisition. Namely, all students, informed and uninformed, know how many students in total are informed. Note that, for uninformed students, knowing or not knowing how many students are informed does not change their strategy. Our overt-information-acquisition approach possibly provides a lower bound on information acquisition regarding one's own preferences. That is, one always has a greater incentive to acquire information covertly and choose to make it public only if she finds it profitable. Besides, the information acquisition is purely about one's own preferences, while all other information is costless.

When no other students are informed and a student acquires this information, the unique equilibrium in the school choice game is:

$$(One) \ Informed : \sigma ((1, 1.1, 0)) = (b, a, c) \ \text{and} \ \sigma ((1, 0.1, 0)) = (a, b, c) ; \\ (Two) \ Uninformed : (a, b, c) ,$$

The informed student obtains an expected payoff:

$$\frac{1}{5}\frac{11}{10} + \frac{4}{5}\left(\frac{1}{3}\left(\frac{1}{10} + 1 + 0\right)\right) = \frac{385}{750}.$$

When she chooses not to acquire information, the game is returned to Scenario (3) and her expected payoff is  $\frac{325}{750}$ . Therefore, given there is no other informed student, her willingness to pay for the information is:

$$w_0^{own} = \frac{385}{750} - \frac{325}{750} = \frac{60}{750}.$$

If there is one informed student already, an additional student acquires this information, and the game has two informed players and one uninformed. The unique equilibrium in this case is:

$$(Two) Informed : \sigma ((1, 1.1, 0)) = (b, a, c) \text{ and } \sigma ((1, 0.1, 0)) = (a, b, c);$$
  
 $(One) Uninformed : (a, b, c).$ 

Informed students obtain an ex ante payoff:

$$\frac{1}{5}\left(\frac{1}{5}\frac{1}{2}\frac{11}{10} + \frac{4}{5}\frac{11}{10}\right) + \frac{4}{5}\left(\frac{1}{5}\frac{1}{2} + \frac{4}{5}\frac{11}{30}\right) = \frac{384.5}{750}.$$

If the student chooses not to acquire information, she plays against one informed and one uninformed players. The equilibrium is discussed above, and her payoff as an uninformed player is:

$$\frac{1}{5}\left(\frac{1}{5}\frac{1}{2} + \frac{4}{5}\frac{21}{30}\right) + \frac{4}{5}\left(\frac{1}{5}\frac{1}{2} + \frac{4}{5}\frac{11}{30}\right) = \frac{335}{750}$$

This implies that the willingness to pay for information in this case is:

$$w_1^{own} = \frac{384.5}{750} - \frac{335}{750} = \frac{49.5}{750}.$$

When the other two students are informed, if the third student also decides to acquire this information, the game turns into one with three informed players as in Scenario (2). We know that her expected payoff is  $\frac{397}{750}$ . If she decides not to do so, she remains uninformed and plays against

two informed players. The equilibrium is discussed above and her expected payoff is:

$$\frac{1}{5} \left( \frac{16}{25} \left( \frac{1}{3} \left( \frac{11}{10} + 1 + 0 \right) \right) + \frac{8}{25} \left( \frac{1}{2} \left( 1 + 0 \right) \right) + \frac{1}{25} \left( 1 \right) \right) \\ + \frac{4}{5} \left( \frac{16}{25} \left( \frac{1}{3} \left( \frac{1}{10} + 1 + 0 \right) \right) + \frac{8}{25} \left( \frac{1}{2} \left( 1 + 0 \right) \right) + \frac{1}{25} \left( 1 \right) \right) \\ = \frac{358}{750}$$

Therefore, the willingness to pay is:

$$w_2^{own} = \frac{397}{750} - \frac{358}{750} = \frac{39}{750}.$$

**Remark 11.** The willingness to pay depends on the number of informed students. When the cost is lower than  $w_2^{own}$ , all students choose to be informed.

**Remark 12.** When more students are informed, the incentive to acquire information is lower.

**Remark 13.** Information acquisition has externalities. Namely, when more students are informed, the payoffs to uninformed students are higher.

**Remark 14.** If we only elicit one amount of willingness to pay, an student reports a number in  $\left[\frac{39}{750}, \frac{60}{750}\right]$ , because she forms a probability distribution over the three possible realizations – playing against another 0-2 informed students.

**The DA Mechanism** Now we consider DA. Students only know the distribution of their own and others' preferences. The following table, Table 14, summarizes the equilibrium strategies and *ex ante* payoffs for informed and uninformed players under DA.

	Table 14. Winnigness to ray for information on Own rayons under DA									
# of Players		Strategy: Strategy: Informed		Ex Ante Payoff		Willingness to				
Informed	Uninformed	Uninformed	(1, 0.1, 0)	(1, 1.1, 0)	Informed	Uninformed	pay for info			
0	3	(a, b, c)	-	-	-	$\frac{325}{750}$	$\frac{5}{750}$			
1	2	(a, b, c)	(a, b, c)	(b, a, c)	$\frac{330}{750}$	$\frac{342.5}{750}$	$\frac{5}{750}$			
2	1	(a, b, c)	(a, b, c)	(b, a, c)	$\frac{347.5}{750}$	$\frac{360}{750}$	$\frac{5}{750}$			
3	0	-	(a, b, c)	(b, a, c)	$\frac{365}{750}$	-				

Table 14: Willingness to Pay for Information on Own Payoffs under DA

When no other students are informed and an student acquires this information, the unique equilibrium in the school choice game is:

(One) 
$$Informed : \sigma ((1,1.1,0)) = (b, a, c) \text{ and } \sigma ((1,0.1,0)) = (a, b, c);$$
  
(Two)  $Uninformed : (a, b, c),$ 

The informed student obtains an expected payoff:

$$\frac{1}{5}\left(\frac{11}{10}\frac{2}{3}\right) + \frac{4}{5}\left(\frac{1}{3}\left(\frac{1}{10} + 1 + 0\right)\right) = \frac{330}{750}.$$

If she chooses not to acquire information, the game is returned to Scenario (3) and her expected payoff is  $\frac{325}{750}$ . Therefore, given there is no other informed student, her willingness to pay for the information is:

$$w_0^{own} = \frac{330}{750} - \frac{325}{750} = \frac{5}{750}.$$

If there is one informed student already, an additional student acquires this information, and the game has two informed players and one uninformed. The unique equilibrium in this case is:

$$(Two) Informed : \sigma((1, 1.1, 0)) = (b, a, c) \text{ and } \sigma((1, 0.1, 0)) = (a, b, c);$$
  
 $(One) Uninformed : (a, b, c).$ 

Informed students obtain an ex ante payoff:

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{1}{2}\frac{11}{10} + \frac{1}{6}\right) + \frac{4}{5}\left(\frac{11}{20}\frac{2}{3}\right)\right) + \frac{4}{5}\left(\frac{1}{5}\left(\frac{1}{2} + \frac{1}{60}\right) + \frac{4}{5}\left(\frac{11}{30}\right)\right) = \frac{347.5}{750}$$

If the student chooses not to acquire information, she plays against one informed and one uninformed players. The equilibrium is discussed above, and her payoff as an uninformed player is:

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{1}{2}+\frac{11}{60}\right)+\frac{4}{5}\frac{21}{30}\right)+\frac{4}{5}\left(\frac{1}{5}\left(\frac{1}{2}+\frac{1}{60}\right)+\frac{4}{5}\frac{11}{30}\right)=\frac{342.5}{750}$$

This implies that the willingness to pay for information in this case is:

$$w_1^{own} = \frac{347.5}{750} - \frac{342.5}{750} = \frac{5}{750}.$$

When the other two students are informed, if the third student also decides to acquire this information, the game turns into one with three informed players as in Scenario (2). We know that her expected payoff is  $\frac{365}{750}$ . If she decides not to do so, she remains uninformed and plays against two informed players. The equilibrium is discussed above and her expected payoff is:

$$\frac{1}{5} \left( \frac{16}{25} \left( \frac{1}{3} \left( \frac{11}{10} + 1 + 0 \right) \right) + \frac{8}{25} \left( \frac{1}{2} + \frac{11}{60} \right) + \frac{1}{25} \left( \frac{2}{3} \right) \right) \\ + \frac{4}{5} \left( \frac{16}{25} \left( \frac{1}{3} \left( \frac{1}{10} + 1 + 0 \right) \right) + \frac{8}{25} \left( \frac{1}{2} + \frac{1}{60} \right) + \frac{1}{25} \left( \frac{2}{3} \right) \right) \\ = \frac{360}{750}$$

Therefore, the willingness to pay is:

$$w_2^{own} = \frac{365}{750} - \frac{360}{750} = \frac{5}{750}$$

**Remark 15.** The willingness to pay is independent of the number of informed students.

Remark 16. Information acquisition has very large externalities.

**Remark 17.** If we only elicit one amount of willingness to pay, an student reports  $\frac{5}{750}$ .

#### **Online.A.6** Scenario (5): (2) + acquisition of information on others' preferences

**The Immediate Acceptance Mechanism** Now suppose everyone knows her own preferences but not others', while the distribution of preferences is common knowledge. With some abuse of terminology, an student is informed if she knows the realization of others' preferences and whether each student is informed or uninformed. An uninformed student knows her own preferences, but neither others' preference realizations nor how many being informed is revealed to uninformed students.

Here, two pieces of information, i.e., other students' preferences and whether they are informed or not, are always acquired together, never separately. As we hypothesize that researching others' preferences is wasteful given independent preferences, we thus study cases where the incentives for wasteful information acquisition is high.

Note that a type-(1, 0.1, 0) student has no incentive to acquire information. Therefore, the discussion of information acquisition is conditional on one's own type being (1, 1.1, 0).

Willingness to pay for information on others' preferences can be similarly defined in the following three cases:

> $w_0^{other}$ : when no other informed students;  $w_1^{other}$ : when there is another informed student;  $w_2^{other}$ : when there are two other informed students.

Table 15 summarizes the equilibrium strategies and *ex ante* payoffs for informed and uninformed players under the Immediate Acceptance mechanism.

When there are no other students informed, the third student can stay uninformed and obtain  $\frac{397}{750}$  ex ante, or  $\frac{681}{750}$  conditional on being type (1, 1.1, 0), as in Scenario (2). If she acquires information on others and becomes informed, the school choice game has the following equilibrium:

$$(Two) Uninformed : \sigma ((1, 1.1, 0)) = (b, a, c); \sigma ((1, 0.1, 0)) = (a, b, c);$$

and the informed player's strategies are summarized in Table 16:

# of	Players	Ex An	te Payoff	Exp. Payoff to Type-(1,1.1,0)		WTP for info				
Informed	Uninformed	Informed	Uninformed	Informed	Uninformed	given type-(1,1.1,0)				
0	3	-	$\frac{397}{750}$	-	$\frac{681}{750}$	$\frac{9}{750}$				
1	2	$\frac{398.8}{750}$	$\frac{396.1}{750}$	$\frac{690}{750}$	$\frac{676.5}{750}$	$\frac{0.6428}{750}$				
2	1	$\frac{396.22857}{750}$	$\frac{398.54}{750}$	$\frac{677.14286}{750}$	$\frac{688.71}{750}$	0				
3	0	$\frac{397}{750}$	-	$\frac{681}{750}$	-					

Table 15: Willingness to Pay for Information on Others' Payoffs under IA

Table 16: Equilibrium Strategies of the Player Informed of Others' Payoffs when Others are Uninformed under IA

Others'	Ex Ante	Action: Info	ormed Player	Ex Post Payo	ff: Informed Player
Preferences	Probability	(1, 0.1, 0)	(1, 1.1, 0)	(1, 0.1, 0)	(1, 1.1, 0)
(1, 0.1, 0) (1, 0.1, 0)	16/25	(a, b, c)	(b, a, c)	11/30	11/10
(1, 1.1, 0) (1, 0.1, 0)	8/25	(a, b, c)	(b, a, c)	1/2	11/20
(1, 1.1, 0) (1, 1.1, 0)	1/25	(a, b, c)	(a, b, c)	1	1

The *ex ante* payoff to the informed player is:

$$\begin{aligned} &\frac{4}{5} \left( \frac{11}{30} \frac{16}{25} + \frac{1}{2} \frac{8}{25} + \frac{1}{25} \right) + \frac{1}{5} \left( \frac{11}{10} \frac{16}{25} + \frac{11}{20} \frac{8}{25} + \frac{1}{25} \right) \\ &= \frac{4}{5} \frac{326}{750} + \frac{1}{5} \frac{690}{750} \\ &= \frac{398.8}{750}. \end{aligned}$$

Therefore, conditional on being type (1, 1.1, 0), the willingness to pay is:

$$w_0^{other} = \frac{690}{750} - \frac{681}{750} = \frac{9}{750}$$

The ex ante payoff to uninformed players, given that there is one informed student, is:

$$\frac{4}{5} \left( \frac{11}{30} \frac{16}{25} + \frac{1}{2} \frac{8}{25} + \frac{1}{25} \right) + \frac{1}{5} \left( \frac{11}{10} \frac{16}{25} + \frac{11}{20} \frac{8}{25} + \frac{11}{20} \frac{1}{25} \right)$$
$$= \frac{4}{5} \frac{326}{750} + \frac{1}{5} \frac{676.5}{750}$$
$$= \frac{396.1}{750}$$

They have no incentives to deviate, and they are worse off than in Scenario (2).

When there is one other student informed, the third student can stay uninformed and obtain  $\frac{396.1}{750}$  ex ante, or  $\frac{676.5}{750}$  when being type (1, 1.1, 0) as above. If she acquires information on others and becomes informed, the school choice game has the following equilibrium in pure strategies:

$$(One) Uninformed : \sigma ((1, 1.1, 0)) = (b, a, c); \sigma ((1, 0.1, 0)) = (a, b, c);$$

and the informed player's strategy is in the following table (Table 17):

Table 17: Equilibrium Strategies with the Informed Player when One of the Others is Informed and the Other is Uninformed under IA

Others' Pre	eferences	Ex Ante	Ante Action: Informed Player		x Ante Action: Informed Player		Ex Post Payor	ff: Informed Player
Uninformed	Informed	Probability	(1, 0.1, 0)	(1, 1.1, 0)	(1, 0.1, 0)	(1, 1.1, 0)		
(1, 0.1, 0)	(1, 0.1, 0)	16/25	(a, b, c)	(b, a, c)	11/30	11/10		
(1, 1.1, 0)	(1, 0.1, 0)	4/25	(a, b, c)	(b, a, c)	1/2	11/20		
(1, 0.1, 0)	(1, 1.1, 0)	4/25	(a, b, c)	(b, a, c)	1/2	11/20		
(1, 1, 1, 0)	(1, 1, 1, 0)	1/25	(a, b, c)	(a, b, c) w/ prob. $6/7^{a}$	1	4/7		
(-,, •)	(-,, -)		(, . , . , . )	(b, a, c) w/ prob. $1/7^{a}$	-			

a. We may allow one informed student to play (a,b,c) and the other informed to play (b,a,c), which is a pure-strategy Nash equilibrium. When either of the two informed students has the same probability to play (a,b,c), the expected payoff of everyone is 31/40 (> 4/7). This leads to a type-(1,1.1,0) student willing to pay 6.75/750 to become informed, given that there is only one more informed student. Moreover, this makes the third uninformed student willing to pay 4.5/750 to be informed. In any case, the interval prediction of WTP for information on others' preferences, which is [0, 9/750] for a type-(1,1.1,0) student, includes all these values.

The *ex ante* payoff to an informed player is:

$$\begin{aligned} &\frac{4}{5} \left( \frac{11}{30} \frac{16}{25} + \frac{1}{2} \frac{8}{25} + \frac{1}{25} \right) + \frac{1}{5} \left( \frac{11}{10} \frac{16}{25} + \frac{11}{20} \frac{8}{25} + \frac{4}{7} \frac{1}{25} \right) \\ &= \frac{4}{5} \frac{326}{750} + \frac{1}{5} \frac{158}{175} \\ &= \frac{396.22857}{750}. \end{aligned}$$

Therefore, conditional on being type (1, 1.1, 0), the willingness to pay given there is another informed agent is:

$$w_1^{other} = \frac{158}{175} - \frac{676.5}{750} = \frac{0.6428}{750}.$$

When there are two other agents are informed, if the third chooses to be informed, we are back to Scenario (1). Conditional on being type (1, 1.1, 0), her payoff is  $\frac{681}{750}$  if being informed. When

two other agents are informed, the third agent, if being uninformed, has a payoff of:

$$\frac{4}{5} \left( \frac{11}{30} \frac{16}{25} + \frac{1}{2} \frac{8}{25} + \frac{1}{25} \right) + \frac{1}{5} \left( \frac{11}{10} \frac{16}{25} + \frac{11}{20} \frac{8}{25} + \frac{46.9}{49} \frac{1}{25} \right)$$
$$= \frac{4}{5} \frac{326}{750} + \frac{1}{5} \frac{688.71}{750} = \frac{398.54}{750}.$$

Therefore,

$$w_2^{other} = \frac{681}{750} - \frac{688.71}{750} < 0.$$

That is, when the other two students are informed, the third student does not have an incentive to acquire information.

**Remark 18.** When only one amount of willingness to pay is elicited, a type-(1, 1.1, 0) student reports a number in  $\left[0, \frac{9}{750}\right]$ . Averaging over all student ex ante, the WTP for information on others's preferences is in  $\left[0, \frac{1.8}{750}\right]$ .

**The DA Mechanism** Since reporting truthfully is a dominant strategy, there is no incentive to know others' preferences.

# Appendix Online.B Analyses of the Game in the Experiment with Risk-Averse Students

This appendix compares risk-neutral and risk-averse students in terms of their willingness to pay for information.

Risk-neutral students have the same cardinal preferences as before (Table 1), and risk-averse students have their von Neumann–Morgenstern utilities associated with each schools as in Table 18.

Iuoie I	Tuble 10. Therefelled a your Tuble for Hisk Twerse Students								
Students	s = a	s = b	s = c						
1	1	$\sqrt{0.1}$ w/ prob. 4/5; $\sqrt{1.1}$ w/ prob. 1/5	0						
2	1	$\sqrt{0.1}$ w/ prob. 4/5; $\sqrt{1.1}$ w/ prob. 1/5	0						
3	1	$\sqrt{0.1}$ w/ prob. 4/5; $\sqrt{1.1}$ w/ prob. 1/5	0						

Table 18: Preference/Payoff Table for Risk-Averse Students

Note that  $\sqrt{0.1} \approx 0.316$ , and  $\sqrt{1.1} \approx 1.049$ . In the following, we evaluate the *ex ante* welfare/payoff, i.e., before the realization of the utility associated with school *b*. Note that ex ante, the expected payoff of being assigned to *b* is  $0.463 \ (\approx \frac{4*\sqrt{0.1}}{5} + \frac{1*\sqrt{1.1}}{5})$  and is better than 1/3 of *a* for any student.<sup>20</sup>

**Conclusion 1.** WTP for own values is smaller for risk-averse students; WTP for others' values is similar when measured as the percentage of expected utilities, but is much lower when measured in dollars.

#### **Online.B.1** Information on Own Values

Willingness to pay can be measured in dollars. However, one dollar does not mean the same in the two cases. Therefore, it is also measured as a percentage of the expected utility under complete information and then of the one under no information.

Table 19: WTP for Info on Own Values: Risk-Averse and Risk-Neutral Students under IA									
# of Other	In D	In Dollars		omplete Info EU	Pctg. of 1	Pctg. of no Info EU			
Informed Players	Averse	Neutral	Averse	Neutral	Averse	Neutral			
0	0.077	0.080	13%	15%	15%	18%			
1	0.062	0.066	11%	12%	12%	15%			
-									
2	0.049	0.052	8%	10%	9%	12%			

Notes: WTP in dollars with risk aversion is calculated as follows: we first obtain the certainty equivalence in dollars of the two expected utilities and then take the difference.

<sup>20</sup>If  $u(x) = \frac{x^{(1-r)}}{1-r}$ , the expected utility from being matched with b is increasing in r which is also the coefficient of relative risk aversion.

In the above table, the complete information expected utility with risk averse under IA is 0.558, while the one with no info is 0.488. The corresponding two expected values for the risk neutral students are  $\frac{397}{750} = 0.529$  and  $\frac{325}{790} = 0.411$ , respectively.

Table 20: Willingness to Pay for Info on Own Values: Risk-Averse and Risk-Neutral Students under DA

# of Other In Dollars		ollars	Pctg. of Complete Info EU		Pctg. of no Info EU	
Informed Players	Averse	Neutral	Averse	Neutral	Averse	Neutral
0	0.003	0.007	0.57%	1.37%	0.61%	1.54%
1	0.004	0.007	0.57%	1.37%	0.61%	1.54%
2	0.004	0.007	0.57%	1.37%	0.61%	1.54%

Notes: WTP in dollars with risk aversion is calculated as follows: we first obtain the certainty equivalence in dollars of the two expected utilities and then take the difference.

#### Online.B.2 **Information on Others' Values**

Note that the willingness to pay for information given one's type being (1, 0.1, 0) is always zero. Therefore, the table below is conditional on the student being type (1, 1.1, 0).

Table 21: WTP for Info on Others' Values: Risk-Averse and Risk-Neutral Students under IA										
# of Other	In Dollars		Pctg. of Complete Info EU		Pctg. of no Info EU					
Informed Players	Averse	Neutral	Averse	Neutral	Averse	Neutral				
0	0.023	0.012	2%	2%	3%	3%				
1	< 0	0.001	-	0%	-	0%				
2	< 0	< 0	-	-	-	-				

Notes: WTP in dollars with risk aversion is calculated as follows: we first obtain the certainty equivalence in dollars of the two expected utilities and then take the difference.

## Appendix Online.C Analyses of Student Strategies/Actions (Rank-Ordered Lists) in the Experiment

This appendix investigates the effects of information provision and the effects of information acquisition on individual strategies. In Appendix Online.A, we derive the equilibrium strategies under various information structure some of which are augmented with information acquisition.

The first information structure is UI (UnInformed: no one is informed about her valuation of school b), under which we have the following hypothesis based on our theoretical results.

**Hypothesis 9** (ROL: UI). A risk neutral player submits a ROL of ABC as a dominant strategy under either IA or DA.

**Result 9** (ROL: UI). When subjects play the game under UI, more subjects play BAC instead of ABC under IA than under DA. Under IA, ABC accounts for 72% of the ROLs, followed by BAC 25%; under DA, 90% play ABC, and 8% submit BAC. The rest plays some other strategies. A session-level Wilcoxon rank-sum (or Mann-Whitney) test rejects the hypothesis that the ABC or the BAC strategy is played equally often under IA and DA (both p-values < 0.01).

Note that the strategy ABC is not a dominant strategy for subjects who are sufficiently riskaverse under IA, which implies that ABC may be less played by more risk-averse subjects. On the contrary, after categorizing the subjects into two almost-equal-sized groups by risk aversion measured in the Holt-Laury lottery choice game, we find that ABC (BAC) are played by 71% (27%) of the less risk-averse subjects who switch choices before or at the 6th Holt-Laury lottery, while ABC (BAC) are played by 77% (21%) of the rest subjects who are more risk averse. This finding is consistent with Klijn et al. (2012) who also show that more risk-averse subjects are not more likely to play "safer" strategies under IA.

Recall that another information structure considered is CI (Cardinally Informed: everyone is informed about her own valuation of school *b* but not others' valuations).<sup>21</sup> Also note that under the treatment of OwnValue, one can acquire information on her own preferences by paying some costs, which results in a game with some informed players and some uninformed. The next hypothesis is about the informed players' strategies. When testing the next hypotheses, the reported p-value is from the session-level Wilcoxon rank-sum (or Mann-Whitney) test, unless noted otherwise.

**Hypothesis 10** (ROL: CI and Acquiring OwnValue). When a subject knows her own preferences but does not know others' preferences, it is a BNE (dominant strategy) to submit a ROL truthfully under IA (DA), regardless of the number of opponents who know their own preferences.

**Result 10** (ROL: CI and Acquiring OwnValue). Under IA, when the valuation of school b is 10, informed subjects are truth-telling at a similar rate -87% with free information, 88% with costly acquired information. When the valuation of school b is 110, there are more subjects playing BAC

<sup>&</sup>lt;sup>21</sup>By design, in this experiment, CI is equivalent to OI (Ordinally Informed: everyone is informed of her ordinal preferences but not others).

with acquired information (90%) than those with free information (85%). However, this difference is not significant (p-value 0.52).

Under DA, when the valuation of school b is 10, informed subjects are truth-telling at insignificantly different rates – 95% with free information, 91% with costly acquired information (p-value = 0.87). When the valuation of school b is 110, however, there are significantly more subjects playing BAC with acquired information (95%) than with free information (79%) (p-value = 0.01).

Lastly, we consider information structure PI (Perfectly Informed: valuations of school b are common knowledge) as a result of information provision and also the OtherValue treatment. Our theoretical prediction regarding the ROL is summarized below.

**Hypothesis 11** (ROL: PI and Acquiring OtherValue). When a subject knows both her own preferences and the preferences of her two opponents, it is a dominant strategy to rank the schools truthfully under DA; the optimal strategy under IA for low-B-valuation subjects report truthfully, while that for high-B-valuation subjects depends on the preference profile as well as the number of informed players.

**Result 11** (ROL: PI and Acquiring OtherValue). Under DA, when the valuation of school b is 10, informed subjects are truth-telling at insignificantly different rates -92% with free information, 84% with costly acquired information (p-value = 0.29). When the valuation of school b is 110, there are fewer subjects playing BAC with acquired information (75%) than with free information (91%). The difference is again insignificant (p-value = 0.86), partly because there are only 16 subjects who successfully acquire information.

Under IA, when the valuation of school b is 10, informed subjects are truth-telling at a similar rate -86% with free information, 84% with costly acquired information. When the valuation of school b is 110, there are insignificantly more subjects playing BAC with acquired information (85%) than that with free information (81%) (p-value = 0.75).<sup>22</sup>

We consider our above results to be consistent with the theoretical predictions. Furthermore, the only case where costly acquired information and freely provided information have significant effects is that when acquired information on OwnValue makes subjects more likely to play the dominant strategy.

<sup>&</sup>lt;sup>22</sup>One may be tempted to investigate subjects' strategies conditional on the preference profile of all subjects. This however makes the samples very small, especially among those who successfully acquire information (61 in total).